Family Risk Sharing, Firm Production Smoothing and the Optimal Size of the Underground Economy.*

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Abstract

This paper shows that informal activities arise not only as a consequence of distortionary taxation, but that there may exist other economic mechanisms behind this phenomenon, like firm's production smoothing and family's risk sharing. The purpose of this paper is to analyze the determinants and the effects of these behaviors with a dynamic general equilibrium model. We show that, with given taxation and institutional parameters, firms' production smoothing and families' risk sharing generate an optimal size for the informal sector.

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1 Introduction.

A large literature exists on causes of the size of the underground economy and on its growth. Among the most quoted factors, there are

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rising of the burden taxes and social security contributions, oppressive regulation in the labor markets, reduction of working time in the official economy and earlier retirement, and difficulties in detecting behaviors involved by such as hidden activities.

This paper discusses two further causes which may generate underground activities. It shows that informal activities arise not only as a consequence of distortionary taxation, but that there may exist other economic mechanisms behind this phenomenon, like the willingness of firms to smooth production and revenues and families' risk sharing. These behaviors are related to the business cycle, and may arise in some circumstances, such as in economies with less developed capital markets and with a traditional family structure characterized by a high degree of institutionalization.

The object of this paper is directly related to the fact that, from the business cycle perspective, the estimated data reveals for many countries that the non-market produced GDP presents cyclical features significantly different from those of the market sector (see Busato and Chiarini 2001). The purpose of this paper is, therefore, to analyze the determinants and the effects of these behaviors with a dynamic general equilibrium model. We show that, with given taxation and institutional parameters, firms' production smoothing and families' risk sharing generate an optimal size for the underground sector.

There are two important reasons why we should be concerned about this result.

First, since these unreported activities make a sizeable contribution to national production and income, we will argue that it is difficult to understand the business cycles without some knowledge of the fluctuations of this relevant component of the aggregate economy. In fact, the model does a good job of replicating the labor market regularities of an European economy with a sizeable underground sector, that are frequently inconsistent with standard real business cycle models. Production smoothing and risk sharing, as defined in this context, augment the standard real business cycle model where resource reallocation occurs inter-temporally.

Second, since the underground economy by definition is not directly observable, and the estimates produced by the econometric methods do not pass the basic statistical tests, a theoretical procedure based on the optimal behavior of the agents and a set of institutional parameters to estimate it should be a valuable way of generating the series *via* stochastic simulations.[1]

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the relationship between underground activities and the phenomena of risk sharing and production smoothing. Section 4 provides numerical results: first, parameters calibration and business cycle implications are discussed, while the section ends with a set of simulation results. Section 5 shows the optimal size of the underground economy and compares it with several data estimations. Data and sources are discussed in Section 6. Section 7 concludes the paper.

2 Structure of the Model.

There are three agents in the model: the firm, the consumer-worker-investor, and the government. In addition there are two sectors: the market and the underground sector. Finally, there is a homogenous consumption good.

The government levies taxes on labor and output (revenues) at constant rates, and does not rebate them back to consumers (wasteful expenditure)[2].

The popular decentralization scheme of the standard stochastic growth model defined by Prescott and Mehra (1980) relies on the infinitely lived consumer-worker-investor and on a one-period-lived firms. We remain within the standard decentralization scheme while giving more content to the firm's side by introducing a intra-temporal dimension that enriches its decision set and enhances the realism of the model.[3] This mechanism relies on the existence of an underground sector cast into a two sector dynamic general equilibrium model, mainly designed for European economies. In particular, the southern European countries characterized by traditional models of family residence, less developed financial markets and sizeable underground sectors match our hypothesis.[4]

[1] See the papers reported in the Economic Journal Symposium on the hidden economy (1999) and Ingram, Kocherlakota and Savin (1997).

[2] Tax burden and social security contributions affect the size of the informal sector positively in all countries. See Tanzi (1999), Giles (1999), Feige (1989) among others. See also Schneider and Enste (2000) for a survey of the literature.

[3] Danthine and Donaldson (2001) propose some interesting alternative approaches to decentralizing the stochastic growth model.

[4] The southern European countries are characterized by a high degree of stability of the family. An institution where different generations are brought together in the

2.1 Production Technologies.

We suppose there a continuum of firms exists uniformly distributed over the unit interval. Firm $i \in [0,1]$ produces a homogenous good using two different technologies, one associated with the market sector, and the other with the non-market sector. Market produced output is denoted as $y_t^{m,i} = M_t \left(k_t^i\right)^{\alpha} (n_t^{m,i})^{1-\alpha}$, and it is the result of capital, k_t^i, and market labor, $n_t^{m,i}$, applied to a Cobb Douglas production function. Underground-produced output equals $y_t^{u,i} = Z_t n_t^{u,i}$, and uses only non-market labor, $n_t^{u,i}$.[5] Finally, M_t and Z_t are two sectoral productivity shocks.

We then assume that in equilibrium each firm allocates a share, θ_t^i, of the total labor demand, n_t^i, to market production (therefore $n_t^{m,i} = \theta_t^i n_t^i$) and the remainder, $1-\theta_t^i$, to the other sector (therefore $n_t^{u,i} = \left(1-\theta_t^i\right) n_t^i$)[6]. The aim is, therefore, to analyze the movement of resources between the two sectors, and to understand how agents want to move inputs out of the market and into the underground. The production technologies become:

$$y_t^{m,i} = M_t \left(k_t^i\right)^{\alpha} \left(n_t^i \theta_t^i\right)^{1-\alpha} \quad \text{and} \quad y_t^{u,i} = Z_t \left(1-\theta_t^i\right) n_t^i. \tag{1}$$

In this context production smoothing is defined as follows:

Definition 1 (Production Smoothing) *Denote as θ_t^i the share of labor demand of $i-th$ firm allocated to the market sector. If $\left(\theta_t^i\right)_{t=1}^{T} \in (0,1)$, in the sense that there are no corner solutions, $i-th$ firm is said*

same household. The family has different implications for the members, where some of them are willing to accept any kind of job in any market, and has the advantage of offering everyone a sort of protection. See the papers in Gallie and Paugam (2000) and the works quoted therein.

[5] Notice that the latter production function is intended as an approximation of a Cobb-Douglas in which capital share, say β, is assumed smaller than the corresponding one in the market sector, α. As Uzawa (1965) and Lucas (1988) prove, if $\beta < \alpha$ we can set the smaller one to zero without loss of any generality. By this result, the structure of production can be simplified, while preserving the main economic intuition.

[6] The use of the share is also consistent both with the fact that labor supply per person is approximately stationary in many economies although the real wage grows, and with the utility function, homogenous in consumption, that we adopt to model the household preferences.

to smooth production, by optimally allocating labor demand between the two sectors on period by period base. [7]

Next, tax-evasion is modelled. Assume that firms pay taxes on market produced revenues at rate $t \in (0,1)$, while they completely evade taxes relative to nonmarket produced revenues. Firms, however, may be discovered by government, with probability $p \in (0,1)$, and forced to pay the tax rate, t, increased by a surcharge factor, $s > 1$. Taking linear projection we compute expected revenues that equal $E_t y_t^i = (1-t) y_t^{m,i} + (1-pst) y_t^{u,i}$ (Lemma 1).

Lemma 1 (Expected Revenues) *Expected revenues are* $E_t y_t^i = (1-t)\, y_t^{m,i} + (1-pst)\, y_t^{u,i}$.

Proof. When a firm is not discovered evading, than its revenues equal $(1-t) M_t \left(k_t^i\right)^{\alpha} \left(n_t^i \theta_t^i\right)^{1-\alpha} + Z_t \left(1-\theta_t^i\right) n_t^i$, which happens with probability $p \in (0,1)$. When, instead, a firm is discovered evading, then its revenues become $(1-t) M_t \left(k_t^i\right)^{\alpha} \left(n_t^i \theta_t^i\right)^{1-\alpha} + (1-st) Z_t \left(1-\theta_t^i\right) n_t^i$, where $s > 1$. That happens with probability $1-p$. Taking linear projection, and simplifying, total expected revenues equal $E_t y_t^i = (1-t) M_t \left(k_t^i\right)^{\alpha} \left(n_t^i \theta_t^i\right)^{1-\alpha} + (1-pst)\, Z_t \left(1-\theta_t^i\right) n_t^i$. ■

The costs to the firm arise from the labor, hired in both sectors, and from renting capital. The cost of market labor is represented by wage paid for hours worked, a wage that is augmented by social security taxes at a rate that we will assume to be equal to the corporate income tax rate, t. In accordance with the rationale behind nonmarket activities, we assume that the firm does not pay social contributions for labor input employed in the nonmarket sector. Summarizing, total costs equal:

$$CO_t = (1+t) n_t^i w_t \theta_t^i + n_t^i w_t (1-\theta_t^i) + r_t k_t^i.$$

The **sectoral productivity shocks,** M_t and Z_t, are specified as a stochastic vector of variables that follow univariate AR(1) processes in log:

$$\begin{pmatrix} \log M_{t+1} \\ \log Z_{t+1} \end{pmatrix} = \begin{pmatrix} \rho_m & 0 \\ 0 & \rho_z \end{pmatrix} \begin{pmatrix} \log M_t \\ \log Z_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1}^m \\ \varepsilon_{t+1}^z \end{pmatrix}$$

[7] It would be interesting to extend the concept of production smoothing intertemporarly. This issue, however, requires a different decentralization framework, where firms live more than one period (see Danthine and Donaldson, 2001).

where ρ_m and ρ_z denote the autocorrelation coefficients of sectoral shocks; finally $[\varepsilon^m_{t+1}, \varepsilon^z_{t+1}]$ denotes a vector of *i.i.d.* random variables.

Firms solve a static optimization problem, and its behavior is characterized by a set of necessary and sufficient first order conditions with respect to k^i_t, n^i_t and θ^i_t. Normalizing n^j_t to unity, rewrite the production functions and the first order conditions, then derive the share of labor demand that is optimally allocated to the market sector, θ^i_t, and the prices supporting the equilibrium, w_t and r_t:

$$\theta^i_t = \left(\frac{(1-t)M_t\left(k^i_t\right)^\alpha(1-\alpha)}{(1-tps)Z_t+w_t t}\right)^{\frac{1}{\alpha}} \tag{2}$$

$$w_t = \frac{(1-t)M_t\left(k^i_t\right)^\alpha(1-\alpha)}{(1+t\theta^i_t)}\left(\theta^i_t\right)^{1-\alpha}+\frac{(1-tps)Z_t}{(1+t\theta^i_t)}(1-\theta^i_t) \tag{3}$$

$$r_t = (1-t)M_t\alpha\left(k^i_t\right)^{\alpha-1}\left(\theta^i_t n^i_t\right)^{1-\alpha}. \tag{4}$$

2.2 Preferences.

Suppose there exist a continuum of families, uniformly distributed over a unit interval, supplying labor to the market and the underground sectors. Family $j \in [0,1]$ has preference over sequences of consumption and labor, and maximizes the following expected utility function:

$$U^j_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c^j_t, \theta^j_t),$$

where E_t is the mathematical expectations operator conditional on information available at time t, and $\beta \in (0,1)$ is a subjective discount factor.

Preference representation accomplishes the existence of two dimensions along which to allocate labor services. Our preference specification is adapted from Cho and Rogerson's (1988) family labor supply and Cho and Cooley's (1994) intensive (the hours worked) and extensive margin (the employment margin) model.[8] Precisely, we assume that $j-th$ family allocates a share θ^j_t of its total labor endowment, n^j_t, to the official labor supply, while the remaining $1-\theta^j_t$ to the underground sector counterpart. We then normalize n^j_t to unity and we specify the momentary utility function as follows:

[8] In our model we reinterpret these two labor dimensions as representing workers' labor supply in the regular and in the underground sectors.

$$u(c_t^j,\theta_t^j)=\frac{(c_t^j)^{1-q}-1}{1-q}-h\frac{(\theta_t^j)^{1+\gamma}}{1+\gamma}(1-\theta_t^j)-f\frac{(1-\theta_t^j)^{1-\eta}}{1-\eta}, \quad (5)$$

where c_t^j denotes the consumption profile of $i-th$ family. This function is separable between consumption and labor and allows us to study how a household allocates its labor between the market and underground sectors. To have a well behaved utility function, we assume that $h,f\geq 0$, $\gamma>-1$, $\eta\in(0,1)$ and that all the parts of the momentary utility function are twice differentiable and well behaved. The second term, $h\frac{(\theta_t^j)^{1+\gamma}}{1+\gamma}(1-\theta_t^j)$, represents the overall disutility of working, while the last term, $f\frac{(1-\theta_t^j)^{1-\eta}}{1-\eta}$, reflects the idiosyncratic cost of working in the underground sector. In particular, this cost may be associated with the lack of any social and health insurance in the underground sector.

The reallocation of hours from the market to the informal sector rather than exclusively from leisure to labor, increases the volatility of the official labor input for a given technology shock.

Feasibility of the program is ensured by the following constraint:

$$w_t(1-\tau_t)\theta_t^j+w_t(1-\theta_t^j)+r_tk_t^j=c_t^j+x_t^j, \quad (6)$$

where x_t denotes investment at time t . Finally, investment increases the capital stock according to a standard state equation: $k_{t+1}^j=(1-\delta)k_t^j+x_t^j$, where δ denotes the depreciation rate.

Risk sharing represents the family side counterpart of production smoothing, and it is defined as follows:

Definition 2 (Risk Sharing) *Denote as θ_t^j the share of labor supply of $j-th$ family allocated to the market sector. Let then $w_t^m=w_t(1-\tau_t)\theta_t^j$ and $w_t^u=w_t(1-\theta_t^j)$ be the market and the underground wage bill, respectively; finally denote as $\rho\left(w_t^m,w_t^u\right)$ the correlation between the two variables. If $\left(\theta_t^j\right)_{t=1}^T\in(0,1)$ (there are no corner solutions), the $j-th$ family is said to smooth its income. Since $\rho\left(w_t^m,w_t^u\right)<0$, the $j-th$ family insures its wage-income profile over the business cycles.*

With the period utility function defined as in equation (5) the value function $\mathcal{J}_t(k_t^j,K_t,\mathbf{A}_t)$ of the $j-th$ family satisfies,

$$\mathcal{J}_t(k_t^j,K_t,\mathbf{A}_t)=\max_{z_t^j,\theta_t^j,n_t^j}\left\{u(c_t^j,\theta_t^j)+\beta E_t\mathcal{J}_{t+1}(k_{t+1}^j,K_{t+1},\mathbf{A}_{t+1})\right\}$$

where $\mathbf{A}_{t+1}$ denotes the vector of stochastic disturbances, $(k_t^j, K_t, \mathbf{A}_t)$ are the state variables for the households in this economy and $(K_t, \mathbf{A}_t)$ are the aggregate state variables. The households solves a dynamic programming problem, maximizing the expected discounted value of the above utility subject to budget constraint (6), and the law of motion for the household capital stock.

After manipulating the necessary and sufficient first order conditions, we obtain the Euler equation (7), and the intra-temporal consumption - labor allocation, condition (8):

$$1 = \beta E_t(\frac{c_{t+1}^j}{c_t^j})^{-q} R_{t+1} \tag{7}$$

$$0 = -w_t \tau_t (c_t^j)^{-q} - (\theta_t^j)^{\gamma} + h\frac{2+\gamma}{1+\gamma}(\theta_t^j)^{1+\gamma} - f(1-\theta_t^j)^{-\eta}, \tag{8}$$

where $((1-t_{t+1})M_{t+1}\alpha \left(K_{t+1}^i\right)^{\alpha-1} \left(\theta_{t+1}^i\right)^{1-\alpha} + 1 - \delta) = (r_{t+1} + 1 - \delta) \equiv R_{t+1}$ from firm profit maximization.

2.3 The Government.

Finally, the flow government budget constraint is:

$$w_t \tau_t \theta_t^j + (pst)\, y_t^u + t y_t^m = G_t, \tag{9}$$

where we are assuming that public expenditure does not affect consumption and production.[9]

2.4 Equilibrium.

The equilibrium for the model presented in previous pages can be characterized as a Variant of the Recursive Competitive Equilibrium (RCE) of the Prescott and Mehra (1980) notion, and it is reported in the Appendix. Specifically, aggregate and individual quantities coincide, and equilibrium can be characterized as the F.O.C. of the representative household on which market clearing conditions have been imposed.

[9]Notice that the government balances its budget only in expectation, since with probability $1-p$ some firms and workers are evading. Hence equation (9) will not be satisfied on a state by state basis.

Before proceeding to the analysis, it should be stressed that the model with an underground sector differs from household (home) production models in several aspects. Precisely, it differs in the commodities' number and their relative substitutability (in the home production class of models there exist two goods, denoted as market and non-market commodities), in the financing of capital investment (in home production models only market-produced goods can be consumed and invested, either into market capital or into non-market capital), and in the insurance opportunities offered by the second sector (an underground sector offers profit smoothing opportunities for firms, and insurance opportunities for consumers). Moreover, the models present different cyclical properties (Ingram et al.,1997, find that hours spent in home production are acyclical).[10]

3 Underground Activities, Risk Sharing and Production Smoothing.

It has been stressed that underground activities in the western countries seem to have been growing over time (see Schneider, 2000). Now, part of this growth can be explained by increasing taxes because individual participation in underground activities at individual as well at aggregate level appears to be positively correlated to tax rates.

At the individual level Clotfelter (1983) considers the impact of total marginal tax rates on evasion, and finds the impact to be positive and significant. He uses data from the 1969 Taxpayer Compliance Measurement Program, and the subsequent literature (surveyed in Andreoni at al. (1998) or Slemrod and Yitzhaki (2002)) has corroborated Clotfelter's results. At the aggregate level, Schneider (2000) shows, using cross country regressions, that there exist a positive, although not overwhelmingly strong, correlation between the size of shadow economy and estimates of total tax burdens.

Notice, however, that effective tax rates on labor, while generally increasing in the OECD countries during the 80s, did not increase in all countries in the 90s. There has been, therefore, a long period along which the size of black economy has continued to grow while tax rates where more or less constant. Hence, there may exist other economic

[10]Fundamental references for the home production models include Greenwood, Hercowitz and Huffman (1988), Benhabib, Rogerson and Wright (1991), Ingram, Kocherlakota and Savin (1997).

mechanisms to explain this pattern. This paper suggests that *firm production smoothing* and *family risk sharing* can give rise to an informal economy.

In what follows Proposition 1 presents firms' behavior for extreme tax rate values, Corollary 1 carries out a monotone comparative statics exercise with respect to tax rates, and Theorem 1 proves that there exists an optimal non-zero size for the underground sector.

Analyze, first, the firm's and the family's choice of allocating labor services (demand and supply) to the non market sector for extreme values of tax rates.[11]

Proposition 1 *Let θ_t^i be the share of regular labor demand, and that θ_t^j of labor supply; it can be showed that:*

1. $t = \tau = 1 \Rightarrow \theta_t^i = \theta_{\min}^i = 0$
2. $t = \tau = 0 \Rightarrow \theta_t^i = \theta_{\max}^i < 1$
3. $\theta_t^j \in (0, 1)\, \forall t$ *and* τ

Proof. To prove part 1 and part 2, substitute $t = \tau = 1$ and $t = \tau = 0$ into equation (2), and solve for θ_t^i. Part 3 is proved numerically, since equation (8) is non linear. The corresponding code is attached in the Appendix. ■

Consider the firm side first. If tax rates were set to unity (i.e. $t = \tau = 1$), no firm allocates labor demand to the market sector, since its whole output is taken away as a tax burden. At contrary, if tax rates are set to zero ($t = \tau = 0$), firms still have incentives to produce using both sectors. This happens because the second sector offers an additional dimension along which firms optimize labor inputs allocation, and by this end, maximize profits.[12] This represents what we have defined before as production smoothing (Definition 1).

Family behavior is slightly more complex to analyze. Households choose how much labor to supply in the market, θ_t^j, and the nonmarket sector, $1-\theta_t^j$, according to equation (8), which is non linear. Hence the value of θ_t^j satisfying that equation has to be derived numerically, after choosing values for the parameters. For the parameter values defined

[11] Notice that here we are proving only the necessary condition. To prove sufficiency it would be necessary to have endogenous taxation.

[12] Notice that these phenomena arise in this model even without distortionary taxation.

in Section 4.1, the share of labor services supplied to the market sector ranges between 8%, when tax rates equal unity (i.e. $t = \tau = 1$), and 70%, when tax rates are set to zero ($t = \tau = 0$).

Corollary 1 completes the picture showing that consumers and families have incentive to shift resources to the underground sector when tax rates increase, still remaining within the boundaries derived in Proposition 1.

Corollary 1 (Monotone Comparative Statics) *Let θ_t^i be the share of regular labor demand, and that θ_t^j of labor supply; it can be showed that $\frac{\partial \theta_t^i}{\partial t} < 0$, $\frac{\partial \theta_t^i}{\partial \tau} < 0$, $\frac{\partial \theta_t^j}{\partial t} = 0$, $\frac{\partial \theta_t^j}{\partial \tau} < 0$.*

Proof. To show that $\frac{\partial \theta_t^i}{\partial t} < 0$ and that $\frac{\partial \theta_t^i}{\partial \tau} < 0$, implicitly differentiate the firm's first order condition for the labor market demand (2) and the family's intra-temporal labor allocation (8) to show that $\frac{\partial \theta_t^j}{\partial t} = 0$ and that $\frac{\partial \theta_t^j}{\partial \tau} < 0$. ■

These results seem to be consistent with both the economic theory and our intuition. An increase in corporate tax rate, t, reduces the share of labor demand allocated to the market sector, while it does not affect the consumer side. An increase in income tax rate, however, reduces the share of labor demand and of labor supply allocated to the regular sector.

In the light of Proposition 1 and Corollary 1, firms might have incentives to work completely in the unofficial sector, while consumers, at least for the parameterization of Section 4.1, always prefer to work in both. In a general equilibrium model market clearing implies that the share of labor demand allocated to the market sector has to be equal to that of labor supply (i.e. $\theta_t^i = \theta_t^j$), and that the same holds for labor services allocated to the non market sector (i.e. $1 - \theta_t^i = 1 - \theta_t^j$). Theorem 1 proves that, in this context, there exists an optimal non-zero share of labor services that is allocated to the non market sector in order to maximize families' utility and firms' profits. This share is denoted as $1 - \theta_t^{**}$.

Theorem 1 (Optimal Size of Underground Sector) *Exist an optimal non-zero size of underground sector, i.e. $\forall (t, \tau) > 0, \exists! \theta_t^{**} \in (0, 1)$.*

Proof. The argument is equivalent to the claim that the solution lies in the interior of the state space. A complete proof is presented in the Appendix. ■

Summarizing, there exist at least two reasons, other than distortionary taxation, which account for the existence of underground activities. These are the production smoothing (Definition 1) on the firm side, and the risk sharing (Definition 2) on the consumer side. Hence the introduction of an underground sector, in which resources are untaxed, enlarges the state-space of the model, while offering to families and firms additional opportunities for allocating resources, improving upon the equilibrium allocation of a standard one-sector model. This intuition is confirmed by the fact that, even if it is feasible to allocate all resources to one sector, agents maximize consumption value and profits using both the sectors. Distortionary taxation plays an important role since it increases or reduces the relative size of an underground sector, but always within certain bounds.

The policy implications are of utmost importance and we discuss them below in Section 5. It can be argued that each country has an optimal size for its black economy that depends on tax rates, on preference parameters, and on production technologies. Section 4 describes the calibration procedure used to simulate the model and provides some business cycle results.

4 Numerical Results.

Being highly non-linear the system has no closed form solution. To study its stochastic properties we apply the well known procedure developed by King, Plosser and Rebelo (1988a, b). In other words, we assume certainty equivalence, linearize the system around its steady state, calibrate parameters, and solve the system applying linear approximations (e.g. Campbell 1994; Cooley 1995; Uhlig 1999).

4.1 Parameters Calibration.

The model is calibrated for the Italian economy though the analysis can be generalized to a large number of European countries which present a sizeable underground sector.[13] Our calibration is based on the seasonally adjusted ISTAT series from 1970:1 to 1996:4, expressed in constant 1995 prices, and on a set of underground output estimations provided by Bovi (1999).

[13]Countries like Belgium, Denmark, Greece , Portugal and Spain have a large share of the underground sector. See, Schneider and Enste (2000).

The system of equations we use to compute dynamic equilibria of the model depend on a set of 16 parameters. Six pertains to household preferences, $(q, h, f, \eta, \gamma, \beta)$, five to the structural-institutional context (the probability of being detected p, the surcharge factor s, the income and corporate tax rates t and τ, and the steady state share of the underground sector, $\bar{\theta}$), and the remaining two parameters to technology (the capital share α, and the capital depreciation rate δ). Finally, $\rho_m, \rho_z, \sigma_m, \sigma_z$ denote the AR(1) coefficients and the standard deviations for the sectoral productivity shocks, respectively.

Table 1: Parameters Calibration

q	η	h	f	γ	α	δ
1.0	0.62	6.0	1.0	2.0	.36	.025
s	p	β	t, τ	$\bar{\theta}$	ρ_m, ρ_z	σ_m, σ_z
1.30	.03	0.98	.275	.735	.95	.712

Table 1. According to the Italian Tax Law (Legislative Decree 471/97, Section 13, paragraph 1) the surcharge s equals 30 or 200 percent of the statutory tax rate. We present results just for the first value. The standard deviations of innovation, σ_m, σ_z, are defined as percentages.

For setting numerical values for parameters, we use the artificial economy as the basis for restricting the general framework and mapping that framework onto the data. Some parameters (α, β, δ) are calibrated so that the non-stochastic steady state of the model matches a set of long-run averages computed from Italian time series (the average value of capital's share of income, the investment-output ratio, the capital output-ratio, the fraction of time households spend working). The structural-institutional parameters (p, s, t, τ) are calculated using information drawn from the number of inspected firms and from the Italian tax Law. Corporate and personal income tax rates are drawn from official data. Preference parameters (h, f, η, γ) are the most difficult to justify. We select them to match several moments concerning the ratios of variances between consumption and production components. The strategy adopted here is, therefore, a sort of signal-extraction procedure. A similar procedure has been used for the parameters of the productivity shocks.

4.2 Selected Business Cycle Implications.

Table 2 presents some summary business-cycle statistics for the Italian economy for the period 1970-1996 and shows that our model fits the basic business cycle moments.

Table 2: Selected Standard Deviations and Correlations

$\sigma(y_t^m)$	$\sigma(y_t^u)$	$\sigma(Y_t^{tot})$	$\sigma(C_t^{tot})$	$\sigma(X_t)$	$\sigma(n_t)$	$\rho(Y_{tot}, \pi_t)$	$\rho(n_t, \pi_t)$
				Actual			
2.27	1.11	**1.44**	**1.25**	3.70	1.39	0.36	0.19
				Simul.			
1.96	0.98	**1.53**	**1.15**	5.59	1.30	0.21	0.25
(0.19)	(0.11)	(0.17)	(0.13)	(0.75)	(0.06)	(0.05)	(0.05)

Table 2. Standard deviation of a variable, say x, is denoted by $\sigma(x)$, while correlation between two variables, say x and y, by $\rho(x, y)$. C_t^{tot} represents ,the consumption of non durable goods and services, Y_t^{tot} is the aggregate GDP, y_t^m denotes the market components, while y_t^u is the underground component. Finally, X_t denotes investment flow and n_t represents market labor input. The statistics are the means of 1000 simulations. Each simulated series is detrended using Hodrick-Prescott filter before the statistics are calculated. The numbers in brackets are small sample standard deviations. *Sources*: C_t^{tot}, X_t, n_t and Y_t^{tot} are withdrawn from Istat database, while y_t^u is from Bovi (1999). The market output component, y_t^m has been obtained using the underground economy estimation y_t^u.

In particular, the model does a good job of replicating the labor market regularities that often are inconsistent with simple RBC models. First, notice that it does not overstate the procyclical nature of average labor productivity: $\rho(Y_{tot}, \pi_t) = 0.21$ in the model and 0.36 in the data. Similarly, it does not overstate the correlation between employment and productivity: $\rho(n_t, \pi_t) = 0.25$ in the model and 0.19 in the data. Second, the model produces relative volatility statistics for employment that are close to those in the data. Third, the model overpredicts the volatility of investment, but the standard deviation of consumption generated by the model is close to the data. [14]

[14] These results are consistent with the widespread empirical evidence that aggregate consumption is smoother than aggregate output, which is one of the most robust empirical evidences matched by equilibrium growth models. Busato and Chiarini (2001) explore in detail how introducing the underground sector in the model goes a long way towards resolving the labor market puzzles.

4.3 Production Smoothing and Risk Sharing: Simulation Results.

Production smoothing and risk sharing represent two further issues for explaining the existence of an underground sector in our framework.

These issues suggest several questions. How are production smoothing and risk sharing related to an underground sector? Do these two phenomena arise only in standard real business cycle models, or also in other classes of models? How do we test for them?

-How are production smoothing and risk sharing related to an underground sector?

To answer the first question, it should be stressed that production smoothing and risk sharing are features peculiar of multi-sector models where outputs (and therefore income sources) produced in different sectors are negatively correlated among them.

This is the peculiar characteristic of the model presented in this paper, where firms and consumers smooth production and wage-income *via* labor demand and labor supply allocation between the two sectors, relying on their strong negative correlation. The negative correlation between outputs produced in the two sectors is, therefore, a feature of the theoretical model aimed to replicate the stylized facts. This characteristic is crucial for firms and consumers to smooth profits, productions and wage income. Notice that, in general, this phenomenon does not arise in a two sector model, where sector components of output present a positive or null correlation. In conventional two-sector models, one would still observe production smoothing, but the volatility of the aggregate series would not be as low as in the case of a negative correlation between output components. Because of the negative correlation with the market production, underground activities can be seen as an *imperfect* substitute for insurance assets.

-Do these two phenomena arise only in standard real business cycle models, or also in other classes of models?

Production smoothing and risk sharing, as defined in this context, augment the standard real business cycle model where resource reallocation occurs just inter-temporally (e.g. King and Rebelo, 1999). Moreover, the mechanism behind production smoothing used by our model differs from a standard linear-quadratic model of inventory behavior, and it also departs from more recent contributions which rely on adjustment of capacity utilization (Wang, 2001), or through changes in the stock of productive capital rather than through finished goods inventories (Chatterjee and Ravikumar, 1993).

-How do we test for them?

We focus on volatility measures for both aggregate and disaggregate outputs generated by the model (and, by this end, of wage incomes), and on correlation among them.

Table 2 (see Section 4.2) shows that the aggregate output, Y_t^{tot}, is less volatile than the market counterpart, y_t^m, whereas the underground output, y_t^u, is smoother than the market and the aggregate variable. Precisely, model simulations provide a standard deviation for the market output of 1.96 whereas the variability of the shadow component amounts to 1.75. By reallocating labor services, firms are able to smooth aggregate output, reducing its standard deviation to 1.36. Since in equilibrium families owe firms, this argument also holds for the families. Thus, a representative family has a twofold source of income, one generated with hours worked in the market sector, and the other with those allocated to underground activities. The proper labor supply allocation ensures risk sharing opportunities, which represent the consumer-side counterpart of production smoothing. These claims are backed by the analysis of correlation matrix for simulated series (**Table 3**):

Table 3: Correlations

	Y_m	Y_u	Y_{tot}	C_{tot}
Actual Data				
Y_m	1.00	-0.81	0.89	0.77
Y_u		1.00	-0.45	-0.54
Y_{tot}			1.00	0.80
C_{tot}				1.00
Simulated Data				
Y_m	1.00	-0.98	0.950	0.70
		(0.01)	(0.01)	(0.11)
Y_u		1.00	-0.96	-0.51
			(0.01)	(0.12)
Y_{tot}			1.00	0.69
				(0.11)
C_{tot}				1.00

Table 3. See Table 2 notes for the notation.

Table 3 emphasizes the cyclical features of the underground economy, showing that both the actual and the simulated data present a negative correlation between underground output and aggregate consumption, whereas this correlation is stronger and positive with market output. The underground sector allows for intra-temporal substitutions of labor, and production smoothing is confirmed by the strong negative correlations between the hidden and the market output.

5 Using Theory for Measurement: Generating Series for Underground Activities.

From the above sections it should be apparent that the presence of a sizeable underground economy distorts many standard economic relationships and affects the cyclical behavior. Underground economy estimates are, therefore, essential for full understanding of economic system and aggregate fluctuations. In policy analysis, it is hard to imagine measures to remove the factors that promote and develop the underground economy without some knowledge of the fluctuations in this nonmarket sector. In general, the policy institutions cannot make major policy changes on the basis of the official data.

In the previous sections, to obtain better information on the aggregate fluctuations we have built a model with two sector and generated cyclical fluctuations taking into account the underground economy phenomenon. Theorem 1 in Section 3 shows that an optimal non-zero size of the underground sector exists. This implies that the theoretical model may be used to infer the size of the unobserved underground sector, which is consistent with the optimal behavior of the representative households and firms. In this work we use a new measurement of the phenomenon.

From the empirical point of view, the difficulty that arise with a non market sector such as the underground economy is that macroeconomic time series data are not available. Often, there exist a number of different estimations in each country. The perception that this economy is sizeable has led a number of authors to attempts to measure its size, influence and implications for the national income and labor input (see, for instance, Bhattacharyya 1999). These authors, often, in carrying out quantitative estimates, do not sufficiently clarify the nature of the phenomenon, considering its complexity.[15]

These factors, along with the necessity of specifying and implementing a vast range of indicators to capture some elements of the unobservable variables, produce troublesome results with the consequence of generating a large gap between estimates over the same period. Of course these results are not valueless. Despite their weakness each estimate of underground economy constitutes a piece of information, and can be useful to improve knowledge of dimension and features of the

[15] This aspect is documented in the Economic Journal Symposium on the Hidden Economy, Vol.109, No 456, June 1999. See also Dallago (1990).

phenomenon. However, the robustness of the various findings is important when, for instance, we want evaluate the structural and the cyclical properties of this behavior and when the government wishes to set out measures which aim to push firms and workers to switch into the market sector. Because of the elusive nature of the underground economy, these estimates may be taken as purely indicative. The reasons for their imprecision and variability should make us cautious when we assess the real significance, in absolute and relative terms, of the size of the irregular sector.

5.1 A Case Study on European Labor Market: the Optimal Size of the Underground Economy in Italy.

In this context, it could be fruitful to utilize a different framework for data construction. In particular, to generate data for unobservable variables which exploit firm and households' optimality. Here we report and compare econometric evidence and model simulations for the hidden sector of the Italian economy.

In the previous sections we derived a set of solutions from a general equilibrium model with an informal sector.[16] Since the model, relying on the optimal intertemporal and intrasectoral behavior of the agents and a set of institutional parameters, is able to infer a set of observations for the underground production and labor force, data construction generated by the model simulations may be a useful source of information and comparison among the estimates of the underground economy. To this end, Figures 1-6 compares the optimal solution in terms of the underground economy achieved with the model, with the existing estimations for Italy.

Italian literature provides a few examples of estimates of macro series, mainly based on the Tanzi methods. In particular, we consider the Bovi's (1999) estimate, which range from 1970 to 1997, and Zizza's (2002) estimate which range from (1984 to 2000). Since we calibrated the model with the statistic properties of Bovi's series for the underground sector, we consider this latter as a benchmark. Finally, Castellucci and Bovi (1999) provides data estimates for the irregular labor input demand for all the Italian regions using information on the local

[16]This method may also be viewed as an improvement of the estimation procedure which, as stressed by Bhattacharyya (1999) and many others, is essential for serious analysis of the interdependence of economic activities.

labor markets (degree of rigidity of the labor markets and probability to be detected).

In comparing these results, we point out the implications of this analysis. Our expressions for labor and underground production are derived from the dynamic general equilibrium model. This produces two consequences. First, our measurement of the underground economy considers both the intertemporal and intratemporal aspects. Second, it is structurally and dynamically consistent with the behavior of the realizations of the model variables (market and nonmarket consumption, investment and capital stock, prices and factor productivity).

Figures 1 and 2 about here

Figures 1 and 2 contain time-series graphs of Bovi's estimated series and the optimal numerical series that we use, overlapped with a Hodrick-Prescott trend curve. It is interesting to note that the data estimation shows a clear downward trend through the middle of the 1970's and the first part of the 1980's, afterward, the trend climbs. The optimal solution for the underground sector rises steadily over the course of the three decades, reflecting a dramatic increase in revenue and income taxation. Despite the exact opposite trend, both the data and the numerical solution of the irregular production show similar cyclical characteristics: the hidden production tends to rise (over the trend) during recessions and decline during expansions. As described in Busato and Chiarini (2001), and Chiarini (2002) market and nonmarket variables are substitutes: the agents are willing to substitute labor, production and consumption over the cycle.

Figure 3 shows the realizations of the model solution and the data for the period 1970-1997. The optimal solution result depends on the particular parameterization of the model. Parameters have not been set arbitrarily, but as discussed above, they follow a careful process for calibrating the model, based on the structural-institutional context and steady-state data information.

Figure 3 about here

It is useful to compare the model prediction for the underground economy to selected estimates. This is done in Figure 3 and 5, respectively, with Bovi's and Zizza's estimations. We have two major findings. First, as noted earlier, although these two estimates are generated by Tanzi's method, they provide quite different results. In these circumstances, a further realization of the unobservable variable consistent with optimal behavior on the part of economic representative agents who can work in both, the market and the hidden sector, could provide a good measure of this phenomenon.[17]

Second, we might wonder whether or not, for a given economic and institutional structure, an optimal underground economy exists and exploit this optimal solution for a comparison with the data. Figure 3 shows a striking result. For a given economic structure and for a determinate taxation (27%), in the 1970's the size of the underground economy was higher than that predicted by the model whereas in the 1980's and 1990's, the hidden production was lower than its optimal value. This would be not the case for lower values of τ and t. Figure 4 shows this numerical solution for the hidden production change with a different calibrated parameters for taxation: $\tau = t = 15\%$ and $\tau = t = 27\%$. An eyeball examination of this graph suffices to evaluate the sizable relationship between taxation and the rise of the underground sector.

Because of these outcomes, we also compare the optimal solution with Zizza's estimation. Figure 5 reports the results for this case and shows that our finding is quite robust. Since the Italian official (corporate and personal income) tax rates are close to the calibrated tax rates, this result has strong implications on the recent policy measures aimed to reduce the underground sector: reducing all distortionary taxation seems to shrink the irregular sector. The figures show that during the last two decades, agents have had significant incentives and willingness to substitute between the market and the underground sector.

Figure 4 and 5 about here

Castellucci and Bovi (1999) also provide underground quantities for the labor input in the Italian regions. Figure 6 reports estimates for four regions. In particular, two northern regions, Liguria and Veneto and two southern regions, Campania e Calabria are compared with the optimal numerical solution (with tax rates 27%). Of course these estimates are achieved with a different approach and, moreover, the model calibration has been set using Bovi's aggregate time series. However, the figure is able to provide new insight into the previous discussion.

[17]Moreover, because complete samples of values for the underground economy are difficult to find, the data generated by the model may be used explicitly in econometric modelling.

By comparing our measurement of nonmarket labor input with the regions data, valuable information arises. Since the underground sizes are remarkably different, a common policy to cope for this sector will fail. The figure reveals that even if a tax cut will produce a reduction in the hidden labor force, the large gap among the local labor markets shows that there would be other elements and incentives that impinge on the firms' and households' willingness to substitute demand and labor supply between the sectors.

Figure 6 about here

6 The Data

Bovi's estimation (1999), is achieved using the currency demand approach. Specifically, the author utilizes an extension of Tanzi's monetary method (1980; 1983). This approach assumes that irregular transactions are undertaken in the form of cash payments. An increase in irregular economy implies an increase in the demand for liquidity. To estimate the excess of currency demand, the equation regressors for currency are chosen so as to include not only standard causes of the money demand (interest rates, habits payments and so on) but also the major factors that may generate the irregular economy phenomenon. Among these, tax and social security burdens are the most important causes. Zizza's estimates used in this work are generated from an application of Tanzi's monetary approach. Moreover, Zizza's (2002), equation regressors allow for a better controlling of the illegal economy, delimiting more precisely the size of the irregular sector. These methods calculate the excess of money setting to zero (or at their minimum level) the "underground factors" which should generate the hidden behavior of firms and households.

Finally, Castellucci and Bovi's (1999) estimates of the irregular labor demand for the Italian regions are derived by utilizing a different indicator approach. This indirect approach aims to investigate the discrepancy between official and actual labor force. The authors' motivations for the irregular labor demand are related to the firms' flexibility (proxied by the unemployment rates), the labor costs and the probability of being detected (expected marginal penalty indicator). Using the estimated long run equilibrium equation with the available regional dataset, the authors obtain a time series of the irregular labor demand for each Italian region.

7 Conclusions

In this paper we argue that production smoothing and risk sharing represent two important behaviors for explaining the existence of an underground sector. We show that, with given taxation and institutional parameters, firms' production smoothing and families' risk sharing provide an optimal size of the informal sector. These behaviors are related to the cycle and may arise more easily in some circumstances, such as in economies with less developed capital markets and with a traditional family structure characterized by a high degree of institutionalization.

Production smoothing as well as risk sharing are, however, not a simple implication of multi-sector models. A necessary condition is that outputs (and thus income sources) produced in different sectors are negatively correlated among them.

Because of the negative correlation with the market production, underground activities can be seen as an *imperfect* substitute for insurance assets. In this context it could be possible to obtain the same volatility of aggregate series only short selling either the market or the underground labor component, in the same spirit of financial hedging.

Finally, since the underground economy by definition is not directly observable and the estimates produced by econometric methods are often not reliable, a theoretical procedure based on the optimal behavior of the agents and a set of institutional parameters to estimate it should be valuable.

8 Appendix

8.1 Definitions and Proofs

Definition 3 (Recursive Competitive Equilibrium) *A Recursive Competitive Equilibrium for the decentralized economy with underground sector consists of:*

1. *a set of* ***continuous price functions****,* $w(\mathbf{A}_t, k_t, K_t) : \Re_2^+ \longmapsto \Re^+$*, and* $r(\mathbf{A}_t, k_t, K_t) : \Re_2^+ \longmapsto \Re^+$*;*

2. *a* ***value function*** $J_t(\mathbf{A}_t, k_t, K_t) : \Re_3^+ \longmapsto \Re^+$*;*

3. *and* ***policy functions*** $c^j(\mathbf{A}_t, k_t, K_t)$*,* $\theta^j(\mathbf{A}_t, k_t, K_t)$*, and* $x^j(\mathbf{A}_t, k_t, K_t)$ *all from* $\Re_3^+ \longmapsto \Re^+$

   such that:

1. ***firms are maximizing profits*** *at the prevailing prices;*

2. $c^j(k_t, K_t, \lambda_t)$ *,* $\theta^j(k_t, K_t, \lambda_t)$ *solve the Euler equation and the intratemporal condition for consumption-labor allocation, i.e.* ***consumer-worker-investors are maximizing utility*** *at the prevailing prices;*

3. ***government balances its budget*** *in expectation, i.e. it holds on a period by period base;*

4. ***market clearing conditions*** *hold for each market. Specifically,*

   for labor service in market sector, $\int \theta^j(\mathbf{A}_t, k_t, K_t)\, dj = \int \theta^i(\mathbf{A}_t, k_t)\, di = \theta_t(\mathbf{A}_t, k_t, K_t) \equiv \Theta_t$

   for labor service in underground sector, $\int (1-\theta^j(\mathbf{A}_t, k_t, K_t))dj = \int (1-\theta^i(\mathbf{A}_t, k_t))di = 1 - \theta_t(\mathbf{A}_t, k_t, K_t) \equiv 1 - \Theta_t$

   for consumption $\int c^j(\mathbf{A}_t, k_t, K_t)\, dj = \int c^i(\mathbf{A}_t, k_t)\, di = c_t(\mathbf{A}_t, k_t, K_t) \equiv C_t$

   for investment $\int z^j(\mathbf{A}_t, k_t, K_t)\, dj = \int z^i(\mathbf{A}_t, k_t)\, di = z_t(\mathbf{A}_t, k_t, K_t) \equiv Z_t$

   for capital $\int k^j(\mathbf{A}_t, k_t, K_t)\, dj = \int k^i(\mathbf{A}_t, k_t)\, di = k_t(\mathbf{A}_t, k_t, K_t) \equiv K_t$

Proof of Theorem 1. There are two possible corner solutions, the first $\theta_t = 0$ is discussed in Claim 1, and the second $\theta_t = 1$ in Claim 2. We prove both by contrapositive argument.

Claim 1: $\theta_t = 0$ **is not a REE**. Suppose not, i.e. $\theta_t = 0$ **is a REE**. If this is the case, it must satisfy profit maximization (equations (4) and

(3)), utility maximization (equations (8) and (7)), and market clearing conditions. From (3) we have that as long as $\theta_t \to 0$ then $w_t \to \infty$. Substituting this into equation (8), the only value of θ coherent with this price is $\theta = 1$. Labor market does not clear: hence $\theta_t = 0$ cannot be a REE.

Claim 2: $\theta_t = 1$ is not a REE. Suppose not, i.e. $\theta_t = 1$ **is a REE**. We use the same argument of Claim 1. From equation (8) when $\theta_t = 1$ then $w_t \to -\infty$ and, from equation (2) there are no values of $\theta_t \in (0, 1)$ that satisfy this condition. Again, market clearing conditions are not satisfied for labor services, and thus $\theta_t = 1$ cannot be a REE. ■

8.2 Code

The solution routine is made of two files: `nonlinearsolver.m` calls the numerical solution algorithm:

```
x0 = [.5]; % Make a starting guess at the solution
options=optimset('Display','iter'); % Option to display output
[x,fval] = fsolve(@theta,x0,options) % Call optimizer
```

Finally, the file `theta.m` contains the functional form of equation to be solved, that is equation (8).

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