

Using Theory for Measurement: an Analysis of the Behavior of Underground Economy*

Francesco Busato¹

Bruno Chiarini²

Vincenzo Di Maro³

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¹ Department of Economics, School of Economics and Management, University of Aarhus

² Corresponding Author. Department of Economics, University of Naples, "Parthenope", Via Medina 40, 80133 Napoli (Italy) Phone: +39.081.5475738; 081.5512207; Fax.: +39.081.5475750, Email : bruno.chiarini@uniparthenope.it.

³ Department of Economics, University College of London.

Abstract.

The underground economy is, by definition, not directly observable, and the estimates produced by the econometric methods do not pass the basic statistical tests. The main difficulty with the existing approaches to the estimation of the underground economy is that they are generally applicable under very restrictive assumptions; in addition the estimation techniques tend not to be statistically robust, providing a large range of estimates of the phenomenon which strongly changes with small changes in the specification. This paper applies the theoretical methodology, first presented by Ingram, Kocherlota, and Savin (1997) for the household production, for generating non-observable series associated to underground economy. Our analysis predicts an underground productivity cycle more volatile than the official estimates; the correlation between the cyclical component of the underground productivity and the GDP cycle is quite different, since official estimates show a positive correlation with the cyclical component of GDP [0.12] while our model's series exhibits a negative and stronger correlation [-0.34].

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1. Introduction

There are two important reasons why we should be concerned about the relationship between underground and regular economy.⁴ **First**, unreported activities make a sizeable contribution to national production and income: for example, it is difficult to understand the business cycles without some knowledge of the fluctuations of this relevant component of the aggregate economy. In fact, models that explicitly incorporate an underground sector, well perform in replicating the labor market regularities of European economies with a sizeable underground sector, which are frequently inconsistent with standard real business cycle models. **Second**, the underground economy is, by definition, not directly observable, and the estimates produced by the econometric methods commonly used in the literature, do not pass the basic statistical tests.⁵

This paper focuses its attention on the second issue.⁶ To obtain knowledge on the underground activities, we require its measurement. To this end, three general sets of measurement approaches may be distinguished in the literature.⁷ The “*direct approaches*” are based on surveys among (supposed) suppliers and demanders of informal services and activities or based on auditing of tax returns undertaken by tax collection and social security administrations. The “*indirect approaches*” share on the assumption that underground transactions are paid in cash in order to make detection more unlikely. The size of the underground economy is reflected in the amount of cash used in a country beyond that used for official transactions. Finally, the “*model approaches*” focus on the causes and effects of the underground economy and try to estimate models for labor, money and product markets using “latent variables” techniques. The models seek to identify the unobserved sector relying on several incentives (burden of taxation and of social security contributions, as well

⁴ There is no agreement over what to include in the underground phenomenon. See, for instance Thomas (1992) or Schneider and Enste, (2000). We refer to this phenomenon as unreported (and therefore untaxed) activities.

⁵ Of course there are many others arguments to investigate on the underground economy. It is clear that the presence of the underground economy distorts many standard economic relations and, therefore, the study of this phenomenon is essential for full understanding of the economic system and for making more effective the policy.

⁶ Busato and Chiarini (2004) and Busato, Chiarini and Rey (2005) investigate the allocation mechanism and the fiscal policy implications related to the first reason in several recent papers.

⁷ See, for instance, the surveys of Schneider and Enste, (2000), Frey B. and Schneider F. (2000) and Oecd (2002; 2004).

as government regulations.) and disincentives to be active in the underground economy (e.g. the probability to be detected and the size of punishment, entailed moral costs). The main difficulty with these approaches is that they are generally applicable under very restrictive assumptions; in addition, the estimation techniques tend not to be statistically robust, providing a large range of estimates of the phenomenon that strongly changes with small changes in specification.

This paper applies the theoretical methodology first presented by Ingram, Kocherlota, and Savin (1997) based on the optimal behavior of the agents (household, firms and a government) for generating the non-observable series associated to underground economy. A theoretical framework is crucial for supporting the “magic numbers” on the size of the underground economy and to help answering several policy questions.⁸ We set out a two-sector dynamic general equilibrium model, in which there are three agents: firms, households and government and one homogeneous good. The government levies proportional taxes on corporate revenues and income flows, and balances its budget in each period. Firms and households, being subject to distortionary taxation, use the underground sector to evade taxes.

We derive equilibrium conditions for underground labor and underground production from a dynamic general equilibrium model. This has the welcome consequences that our measurement of the underground economy incorporates both an inter-temporal and an intra-temporal aspects

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the traditional methods used by the existing literature to estimate underground activities. Section 4 suggests a complementary approach: “using theory for measurement”, in the spirit of Ingram, Kocherlota, and Savin (1997). Next, Section 5 shows the empirical results whereas Section 6 performs a sensitivity analysis for a set of parameters of the model. Section 7 concludes the paper. Data and sources are discussed in the Appendix.

⁸ See, for instance, Thomas (1999) and Giles (1999) works. The former author stresses the “heroic assumptions” of the approaches used to estimate the unrecorded sector, whereas Giles provide a more mild criticism to the econometric techniques to measure the size of the underground economy.

2. Model's structure

The economy is populated with a large number of identical households, each of which will live forever and each with identical preferences defined over consumption and labor inputs at every date. Each household allocates labor services in the regular sector and in the informal sector. The economy is also populated by a large number of identical firms each of which rent capital and labor services from the households.

2.1 Households preferences

There is a continuum of agents indexed by $\gamma \in [0,1]$ with preferences over consumption and labor services. We restrict the momentary utility function to have the following form:

$$U_t^\gamma = \frac{(c_t^\gamma + \varphi c_{G,t})^{1-q} - 1}{1-q} - B_M \frac{(n_t^\gamma)^{1+\xi}}{1+\xi} - B_U \frac{(n_{U,t}^\gamma)^{1+\psi}}{1+\psi}, \quad (1)$$

where c_t^γ is the private consumption, $c_{G,t}$ stands for government purchases of the homogeneous good produced in the economy, n_t^γ is the total labor services supply and $n_{U,t}^\gamma$ is the underground labor supply. Notice that $B_U \geq 0; B_M \geq 0$. The last term $B_U \frac{(n_{U,t}^\gamma)^{1+\psi}}{1+\psi}$ is the idiosyncratic cost of working in the underground economy, and the term $B_M \frac{(n_t^\gamma)^{1+\xi}}{1+\xi}$ stands for the overall disutility of working.

There are two resource constraints, the chosen working time which must be allocated to the regular and underground sector and the total use of goods for consumption and investment cannot exceed the disposable income net taxes:

$$\begin{aligned}
n_{M,t}^\gamma + n_{U,t}^\gamma &\leq n_t^\gamma \\
c_t^\gamma + i_t^\gamma &= (1 - \tilde{\tau}_t^\gamma) (w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma) + w_{U,t} n_{U,t}^\gamma \\
k_{t+1}^\gamma - (1 - \Omega) k_t^\gamma &= i_t^\gamma,
\end{aligned} \tag{2}$$

where Ω denotes a quarterly depreciation rate for private capital stock. In addition, the households own an initial stock of capital, $k_{i,0}^\gamma$, which they rent to firms and may augment through investment.

The household's problem is the following:

$$\begin{aligned}
&\max_{\{c_t^\gamma, i_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma, n_t^\gamma\}_{t=0}^\infty} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma, n_t^\gamma; c_{G,t}, \Xi) \\
&s.t. \\
&c_t^\gamma + i_t^\gamma = (1 - \tilde{\tau}_t^\gamma) (w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma) + w_{U,t} n_{U,t}^\gamma \\
&i_t^\gamma = k_{t+1}^\gamma - (1 - \Omega) k_t^\gamma \\
&n_{M,t}^\gamma + n_{U,t}^\gamma = n_t^\gamma \\
&k_{i,0}^\gamma, c_{G,t} \text{ given}; c_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma, n_t^\gamma > 0 \\
&\tilde{A}_{t+1} = dF(\tilde{A}_{t+1}; \tilde{A}_t),
\end{aligned} \tag{3}$$

where E_0 is the mathematical expectation operator (at $t=0$), Ξ stands for the parameter's set and $\tilde{A}_t = [\tilde{\tau}_t^\gamma, \tilde{\tau}_t^\pi, \lambda_t]$ is a stochastic disturbances vector that will be discussed below. Consumers take government purchase of final commodities, $c_{G,t}$, as given.

The representative consumer choice leads to the following equations:

$$n_{U,t}^\gamma = \left(\frac{w_{U,t} - (1 - \tilde{\tau}_t^\gamma) w_{M,t}}{(1 - \tilde{\tau}_t^\gamma) w_{M,t}} \right)^{\frac{1}{\psi}} \left(\frac{B_M}{B_U} \right)^{\frac{1}{\psi}} (n_t^\gamma)^{\frac{\xi}{\psi}} \tag{5}$$

$$(c_t^\gamma + \varphi c_{G,t})^q = \left(\frac{w_{U,t} - (1 - \tilde{\tau}_t^\gamma) w_{M,t}}{B_U (n_{U,t}^\gamma)^\psi} \right) \tag{6}$$

The equation (5) shows that the underground labor supply depends on the total labor supply n_t^γ , the ratio between the two inverse elasticities (ξ/ψ) and the difference between the wage rates.

Equation (6) shows that the aggregate consumption is determined by the tax wedge, which, in turns, depends on the income tax $\tilde{\tau}_t^Y$. Condition (6) suggests that the larger the tax wedge is, the more resources are allocated to the underground sector. Households may finance increases in consumption by raising their underground income by working in the underground sector.

Optimal investment choice depends on the following Euler equation:

$$(c_t^\gamma + \varphi c_{G,t})^{-q} = \beta E_t (c_{t+1}^\gamma + \varphi c_{G,t+1})^{-q} ((1 - \tilde{\tau}_{t+1}^Y) r_{t+1} + 1 - \Omega) \quad (7)$$

The economy wide constraint is given by

$$C_t + I_t + G_t = Y_t, \quad (8)$$

where G_t is the aggregate government spending; aggregate variables are defined as $C_t = \int_0^1 c_t^\gamma d\gamma$;

$$I_t = \int_0^1 i_t^\gamma d\gamma; Y_t = \int_0^1 y_t^i di.$$

2.2 Production technologies

Each firm $i \in [0,1]$ uses employment supplied to the regular labor market and capital to produce the regular output $y_{M,t}^i$, whereas employment supplies to the informal sector is used to produce underground output $y_{U,t}^i$:⁹

$$y_{M,t}^i = \lambda_t (k_t^i)^\alpha (n_{M,t}^i)^{1-\alpha} : y_{U,t}^i = \lambda_t (n_{U,t}^i) \quad (9)$$

The production technologies are scaled by λ_t , an exogenous productivity shock which is able to alters the average level of output.

⁹ This technology specification is proposed in Busato and Chiarini (2004) and it can be show that is derived by a more general set-up where both the production functions use capital and labor inputs.

2.3 A model of tax evasion

Firms may be discovered evading with probability $p \in (0,1)$ and forced to pay the stochastic tax rate τ_t^Π , increased by a surcharge factor $s > 1$. The following scheme summarizes the tax evasion model:

$$REV_t^i \begin{cases} \rightarrow \text{Detected}(\sim p) & REV_{D,t}^i = (1 - \tau_t^\Pi)y_{M,t}^i + (1 - s\tau_t^\Pi)y_{U,t}^i \\ \rightarrow \text{Not Detected}(\sim 1-p) & REV_{ND,t}^i = (1 - \tau_t^\Pi)y_{M,t}^i + y_{U,t}^i \end{cases} \quad (10)$$

In order to compute the expected revenues, we apply linear projection:

$E_t REV_{D,t}^i = pREV_{D,t}^i + (1-p)REV_{ND,t}^i$, which yields:¹⁰

$$E_t REV_{D,t}^i = (1 - \tau_t^\Pi)y_{M,t}^i + (1 - ps\tau_t^\Pi)y_{U,t}^i \quad (11)$$

2.4 Firm behavior

At each date t , firm i maximizes period expected profits¹¹:

$$\begin{aligned} \max_{\{n_{M,t}^i, n_{U,t}^i, k_t^i\}} \tilde{\Pi}_t^i &= E_t REV_t^i - w_{M,t}n_{M,t}^i - w_{U,t}n_{U,t}^i - r_t k_t^i \\ s.t. & \\ y_{M,t}^i &= \lambda_t (k_t^i)^\alpha (n_{M,t}^i)^{1-\alpha} : \quad y_{U,t}^i = \lambda_t (n_{U,t}^i) \\ E_t REV_{D,t}^i &= (1 - \tau_t^\Pi)y_{M,t}^i + (1 - ps\tau_t^\Pi)y_{U,t}^i \\ n_{U,t}^i > 0; n_{M,t}^i > 0; k_t^i > 0 \end{aligned} \quad (12)$$

The following first order conditions characterize the corporate efficiency conditions:¹²

$$\begin{aligned} w_{M,t} &= (1 - \tilde{\tau}_t^\Pi)(1 - \alpha)(k_t^i)^\alpha (n_{M,t}^i)^{-\alpha} \lambda_t \\ w_{U,t} &= (1 - ps\tilde{\tau}_t^\Pi)\lambda_t \end{aligned} \quad (13)$$

¹⁰ Where the following “no bankruptcy condition” is imposed, $(1 - ps\tau_t^\Pi) \geq 0$.

¹¹ In Prescott and Mehra’s (1980) recursive equilibrium scheme, firms solve a myopic profit maximization problem on a period-by-period basis. See also the papers included in Cooley (1995).

¹² It can be shown that when there is no distortionary taxation equilibrium underground labor services equal zero, as long as underground sector average productivity is sufficiently low. This represents the trade-off between employing professional labor services on the regular market (and paying taxes on it), and employing less productive labor on the underground sector (with the advantage of evading the associated taxes). In this case underground labor services are used for evading taxes, even if those services have a relatively lower productivity.

$$R_t = (1 - \tilde{\tau}_t^\Pi) \alpha (k_t^i)^{\alpha-1} (n_{M,t}^i)^{1-\alpha} \lambda_t$$

2.5 Government

The government is described as a sequence $\{\pi_t\}_{t=0}^\infty$ of tax rates on consumers' income and on firm's profits:

$$\{\pi_t\}_{t=0}^\infty = \{\tilde{\tau}_t^Y, \tilde{\tau}_t^\Pi, \tilde{G}_t\}_{t=0}^\infty \quad (14)$$

Government collects tax revenues $R\tilde{V}_t$:

$$R\tilde{V}_t = E_t \left[(w_{M,t} n_{M,t}^i + r_t k_t^i) \tilde{\tau}_t^Y + \tilde{\tau}_t^\Pi (y_{U,t} ps + y_{M,t}) \right]$$

In this model both consumers and firms evade taxes. Consumers evade income taxation producing an income loss associated to the underground produced income flow $w_{U,t} n_{U,t} \tilde{\tau}_t^Y$. Firms always try to evade an amount of taxes equal to $\tilde{\tau}_t^\Pi y_{U,t}$. When discovered, firms pay the additional fine $\tilde{\tau}_t^\Pi y_{U,t} ps$. That happens, as described above, with probability p . In the other case, firms do evade and are not discovered with probability $(1-p)$. The expected corporate tax evasion equals $(1-p)\tilde{\tau}_t^\Pi y_{U,t}$. Total tax evasion is, therefore:

$$E_t TE_t = \left[(1-p)\tilde{\tau}_t^\Pi y_{U,t} + \tilde{\tau}_t^Y w_{U,t} n_{U,t} \right]$$

Government spending is allocated to purchase of final consumption goods and it equals government revenues:

$$\tilde{G}_t = R\tilde{V}_t = c_{G,t} \quad (15)$$

2.6 Stochastic Disturbances

$$A_{t+1} = QA_t + v_t, \quad (16)$$

where $A_t = [\tilde{\tau}_t^Y, \tilde{\tau}_t^\Pi, \lambda_t]$ and v_t is a vector including shocks' innovations; the autocorrelation coefficient matrix Q and the covariance matrix Σ are defined below:

$$Q = \begin{bmatrix} \varphi_\lambda & 0 & 0 \\ 0 & \varphi_Y & 0 \\ 0 & 0 & \varphi_\Pi \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_{v,\lambda} & 0 & 0 \\ 0 & \sigma_{v,Y} & 0 \\ 0 & 0 & \sigma_{v,\Pi} \end{bmatrix}$$

2.7 Competitive Equilibrium Characterization

A competitive allocation is a policy $\{\pi_t^*\}_{t=0}^\infty$, an allocation $\{x_t^*\}_{t=0}^\infty = \{K_t^*, C_t^*\}_{t=0}^\infty$ and a price system

$\{1, w_{M,t}^*, w_{U,t}^*, r_t^*\}_{t=0}^\infty$ such that, given the policy and the price system, the resulting allocation

maximizes the representative consumer utility (condition (4)) subject to *i*) the sequence of budget constraints (condition (2)); *ii*) the price system $\{1, w_{M,t}^*, w_{U,t}^*, r_t^*\}_{t=0}^\infty$ (conditions (13)); *iii*) the government budget constraint is satisfied on average (condition (15)); *iv*) and the market clearing conditions hold for each market (condition (8)).

3. Classical Methods for Estimating Size of the Underground

Economy

Underground economy estimates represent important information for a complete understanding of an economic system, of its fluctuations, and of its response to macroeconomic policies, among the many issues.

The perception that the underground economy is sizable led a number of authors to attempts to measure its size, influence and implications for the national income and labor input (see, for instance, Bhattacharyya, 1999). The authors, often, in carrying out quantitative estimates, do not sufficiently clarify the nature of the phenomenon, because of its complexity.¹³ These difficulty, along with the necessity of specifying and implementing a vast range of indicators to capture some elements of the unobservable variables, bring to troublesome results with the consequence of

¹³ This aspect is documented in the Economic Journal Symposium on the Hidden Economy, Vol.109, No 456, June 1999. See also Dallago (1990), Thomas (1992), and Chiarini and Marzano (2004).

generating a large gap between estimates over the same period. Of course these results are not valueless. Despite their weakness each estimate of underground economy constitutes a piece of information, and can be useful to improve knowledge of dimension and features of the phenomenon. However, the robustness of the various findings is important for evaluating the structural and the cyclical properties of the underground sector's behavior, and when the government wishes to set out measures which aim to push firms and workers to switch into the regular sector. Because of the elusive nature of the underground economy, the estimates provided by the econometrics literature may be taken as purely indicative. The reasons for their imprecision and variability should make us cautious when we assess the real significance, in absolute and relative terms, of the size of the irregular sector. In this context, it could be fruitful to utilize a different framework for data construction. In particular, a new approach which exploits firms and households' optimality to generate data for unobservable variables.

In addition, the main empirical difficulty is that sufficiently long macroeconomic time series data are not available; furthermore, there exist a number of different estimates in each country.

Here comes our contribution. In order to obtain information on the aggregate fluctuations we use the previously presented two-sector model for generating the unobserved data. The use of a general equilibrium model to infer the size and the dynamics of an unobserved underground sector ensures the consistency with the optimal behavior of the representative households and firms.

3.1 A Case study on European labor market: the size of the underground economy in Italy.

Figures 1 compares several existing estimates of the Italian underground economy obtained with different empirical approaches. Italian literature provides a few examples of estimates of macro series, mainly based on the Tanzi methods. In particular, we consider the Bovi's (1999) estimates, which range from 1970 to 1997, Zizza's (2002) estimates which range from 1984 to 2000, Chiarini and Marzano's (2004) estimates which range from 1976 to 1998 and the official National Institute for Statistics (Istat) estimates for the period 1992 to 2003.

Here we want to stress a major finding. As noted earlier, although many of these estimates are generated by Tanzi's methods, they provide for the same periods quite different results in terms of size and dynamics of the informal sector. In these circumstances, a different approach to the estimation of the unobservable variables which is consistent with optimal behavior on the part of economic representative agents who are able to operate in both, the regular and the underground sector, could provide a good measure of this phenomenon and, in any case, a useful benchmark.¹⁴

4 “Using theory for measurement”

We show that our model, relying on the optimal intertemporal and intrasectoral behavior of the agents and a set of institutional parameters, is capable to generate a set of observations for the underground production and labor force. Although we do not use statistical inference (observed data to draw conclusions about the population from which the data came), the data and the process that generates them may be useful sources of information and comparison among the various estimates of the underground economy.

4.1 The Ingram, Kocherlota, Savin (1997) methodology

The first application of the methodology we use here is due to Ingram, Kocherlota and Savin (1997) (IKS). They study the cyclical behavior of home production under the motivation that without a good knowledge of the home sector it is not possible to fully understand the business cycle. Their seminal approach follows from the fact that data for home-sector activities is not available at high frequencies (at best is available yearly). In brief, they use macroeconomic theory in conjunction with aggregate macroeconomic data to infer the behavior of unobserved home-sector variables.

In particular, IKS consider a representative agent with stable preferences but stochastic production technologies in the home and market sectors. Then, they assume that observed market

¹⁴ Moreover, because complete samples of values for the underground economy are difficult to find, the data generated by the model may be used explicitly in econometric modelling.

variables are well measured and consistent with the optimal behavior of this representative agent. Accordingly, they use the representative agent's first order conditions to generate quarterly series for the home-sector variables. We specialize here this approach to the measurement of the underground sector.¹⁵

4.2 Derivation of the optimal behavior condition

According to our model, the representative agent makes optimal choices at every point in time taking into account both the regular and the underground sector. Data can be directly informative only about the variables of the observed regular sector. However, in general it is possible to infer the behavior of the underground sector variables using the optimality conditions from the theory in conjunction with data, as we illustrated above.

In particular, here we derive equilibrium condition characterizing the optimal labor allocation to the underground sector. This optimality condition, next, can be used to generate data for the underground sector. We combine the representative optimality condition (5) and the corporate efficiency condition $w_{U,t} = (1 - ps\tilde{\tau}_t^\Pi)\lambda_t$ to obtain:

$$\frac{n_{U,t}^\gamma}{n_t^\gamma} = \left(\frac{(1 - ps\tilde{\tau}_t^\Pi)}{(1 - \tilde{\tau}_t^Y)w_{M,t}} - 1 \right)^{\frac{1}{\psi}} \left(\frac{B_M}{B_U} \right)^{\frac{1}{\psi}} (n_t^\gamma)^{\frac{\xi}{\psi} - 1} \quad (17)$$

At every point time firms' optimality conditions imply that the wage in the regular sector can be expressed as $w_{M,t} = (1 - \tilde{\tau}_t^\Pi)(1 - \alpha)(k_t^i)^\alpha (n_{M,t}^i)^{-\alpha} \lambda_t$. This equation can be rewritten imposing also the normalization restriction $\lambda_t = 1$, as follows:¹⁶

$$w_{M,t} = (1 - \tilde{\tau}_t^\Pi)(1 - \alpha) \frac{y_{M,t}^i}{w_{M,t}} \quad (18)$$

This allows us to write the labor-input ratio as follows:

¹⁵ A comparison between the underground economy and the home production model's characteristics is reported in Appendix A.3.

¹⁶ Notice that in our model the scale productivity parameter λ_t enters both the production function of the regular and underground sector in the same fashion.

$$\frac{n_{U,t}^\gamma}{n_t^\gamma} = \left(\frac{(1 - ps\tilde{\tau}_t^\Pi)}{(1 - \tilde{\tau}_t^Y)(1 - \tilde{\tau}_t^\Pi)(1 - \alpha) \frac{y_{M,t}}{w_{M,t}}} - 1 \right)^{\frac{1}{\psi}} \left(\frac{B_M}{B_U} \right)^{\frac{1}{\psi}} (n_t^\gamma)^{\frac{\xi-1}{\psi}}. \quad (19)$$

Equation (19) expresses the share of underground employment (out of total employment) as a function of total employment n_t^γ , productivity in the regular sector $\frac{y_{M,t}}{w_{M,t}}$, preference and taxation parameters. An implication of (19), for example, is that whenever the total employment and the productivity are at their equilibrium values, the share of underground employment will be as its equilibrium value as well. On the other hand, if one variable is out of its equilibrium value then the others will adjust to keep the equality in (19).

Equation (19), moreover, is an equilibrium relationship, and it does not necessarily hold if one (or both) of the series in the right-hand side exhibit a trend. Accordingly, we apply the Hodrick-Prescott filter to the series for the regular sector productivity and for the total employment and use detrended series in the generation of the unobserved share of underground employment series.¹⁷ The detrended series will fluctuate around zero with departures from equilibrium expressed in the specific series' units (for example for the total employment the disequilibrium will be expressed in number of labor units in excess, or below, of the equilibrium value). To make the series consistent with a common metric of disequilibrium we implement a two step transformation: first, we assume that the mean of each series is a proxy of its equilibrium value. Then, we transform the series in order to have as equilibrium value 1. In brief, the result of this strategy is that the series for total employment and the productivity in the regular sector take value 1 when they are at their equilibrium value (the mean) and values expressed as percentage of the mean when out of the equilibrium.

¹⁷ See the Appendix for details on the data set.

To summarize, our procedure for measuring the underground sector consist of two main parts: first, we use prediction from theory. In particular, we use the representative consumer's optimality conditions and the corporate efficiency conditions to derive an equation that expresses the underground unobserved sector variable (in our case the underground employment quota) as a function of only observed variables. The second step is to use the observed data to generate the underground sector variable. This step involves also to re-scale the observed series in order to make them consistent with the fact of (19) being an equilibrium condition.

It is important to stress that this methodology is robust to changes of the context in which one applies it. Here we apply the procedure to generate a particular kind of unobserved variable, the employment in the underground sector. However, the same methodology could be applied to generate any kind of unobserved series as long as there is a model that delivers usable predictions.

5. Empirical Results

We generate series for the share of employment in the underground sector (see Figure 2) using the condition (19) and the parameterization of the model reported in Table 1. Preference and technology parameters are consistent with the canonical literature: logarithmic utility ($q=1$); $\beta=1$, $\alpha=0.33$ and $\Omega=0.025$. Labor supply parameters follow the analysis of Blundell and MaCurdy (1999), whereas the disutility parameters are calibrated to match volatilities of aggregate labor and investment. For the probability of being detected (p) and the penalty factor (s) we refer to Busato and Chiarini (2004) who calibrate these parameters using the information on the number of inspected firms.¹⁸

[Here FIGURE 2]

¹⁸ We may wonder if the settled parametrization provides business cycle statistics able to replicated real data. The numerical results of simulating this economy are shown in Appendix A.2.

Table 1 – Parameterization of the model

τ^y	τ^π	α	Ω	p	s	ψ	B_M	B_U	q	ξ
0.3436	0.4155	0.33	0.025	0.03	1.3	0.06	1.2	1.4	1	1

A generated series for the share of underground GDP is also provided in Figure 3. This series is constructed using the underground sector production technology function as in (10)¹⁹.

[insert FIGURE 3]

Table 2 reports selected descriptive statistics for our two generated series and for the selected other series that estimate Italian underground economy, discussed in Section 3 (see Figure 1). Notice that our generated series are quarterly while all the other series are yearly and span different time periods (see details in Table 2). This makes comparison quite difficult but does not rule out the possibility of highlighting some general pattern. First, our generated series are more volatile than all the other series and they span a range consistently higher (25.6 and 23.4 for, respectively, underground employment and GDP share while the highest other series' range is Bovi's with 6.39). The model normalized standard deviation of the underground production is remarkable large compared with the estimated series. Second, there is some evidence of a higher persistence of our generated series since the first autocorrelation is consistently higher the other series, even though Istat's (version min) shows a quite high autocorrelation (0.798). Third, all the series appear to be pro-cyclical: they are positively correlated with the Hodrik-Prescott (1997) filtered GDP.

¹⁹ The series of the underground GDP shares the same characteristics of the series of the underground employment as a result of the particular production technology in the underground sector assumed by our model. Accordingly, all the following analyses are conducted on the underground employment series.

Table 2 – Comparisons with existing estimates of underground sector

Statistics	$\left(\frac{n_{U,t}^\gamma}{n_t^\gamma}\right)^*$	$\left(\frac{y_{U,t}^\gamma}{y_t^\gamma}\right)^*$	Chiarini Marzano_1	Chiarini Marzano_2	Istat GDP max	Istat GDP Min	Istat ULA	Bovi	Zizza
Numb. Of Obs	128	128	23	23	9	9	23	28	16
Mean	22.1	21.6	22.5	7.7	16.8	15	6.0	14	16.5
Min	13.1	12.8	19.5	6.9	15.8	12.9	5.1	11.7	14.8
Max	38.7	36.2	25.6	8.6	17.7	15.9	6.5	18.1	18.7
Range	25.6	23.4	6.1	1.7	1.9	3	1.4	6.39	3.89
σ_{series}	0.227	0.187	0.080	0.062	0.030	0.070	0.063	0.136	0.065
$\frac{\sigma_{series}}{\sigma_{GDP}}$	19.730	16.253	6.954	5.388	2.619	6.091	5.475	11.847	5.675
$\rho(series, GDP)$	0.548	0.467	0.634	0.645	0.15	0.392	0.84	0.459	0.726
$\rho(t, t-1)$	0.904	0.842	0.910	0.910	0.089	0.798	0.861	0.545	0.568

Frequency and period: $\left(\frac{n_{U,t}^\gamma}{n_t^\gamma}\right)^*$, $\left(\frac{y_{U,t}^\gamma}{y_t^\gamma}\right)^*$ quarterly, 1970:1-2001:4; Chiarini, Marzano 1, yearly, 1976-1998;

Chiarini, Marzano 2, yearly, 1976-1998; Istat max, Istat min, yearly, 1992 – 1998; Istat ULA, yearly, 1980- 2002; Bovi, yearly, 1970 – 1997; Zizza, yearly, 1984 – 2

A further interesting result arises when we examine the plot of our generated underground employment ratio and the Hodrik-Prescott (1997) filtered GDP (see Figure 4). Assuming a procyclical regular employment, we note that also the underground labor input shows a pro-cyclical pattern. In addition to the employment series being pro-cyclical, inspection of Figure 4 allows us to deduce that underground employment seems to follow filtered GDP with some lag. This deduction is confirmed by the values of the correlation between the filtered underground employment variable and the lagged filtered GDP [$\rho(series, GDP_{t-i}) = 0.548 (i=0), 0.6698(i=1), 0.6995 (i=2), 0.6485 (i=3), 0.5182 (i=4)$]. Thus, correlation with the 1-period lagged filtered GDP is higher than the contemporaneous correlation.

[insert: FIGURE 4]

5.1 Productivity in the underground sector

We plot the series of the underground productivity generated by our model and the filtered GDP against time in Figure 5.

[insert: FIGURE 5]

[insert: FIGURE 6]

Since Istat provides a yearly estimate of GDP and employment in the underground sector starting from 1992, an estimate of the productivity in the underground sector can be deduced from these official estimates. Figure 6, next, plots both our model's and Istat's version of the underground productivity series both filtered using HP approach.

One clear point is that our model predicts an underground productivity cycle more volatile than Istat's (the model's standard deviation is 0.0042 whereas Istat's statistics is 0.0019). It is also interesting to note that the correlation between the cyclical component of the underground productivity and the GDP cycle is quite different, since Istat's series shows a positive correlation with the cyclical component of GDP [0.12] while our model's series exhibits a negative and stronger correlation [-0.34]. This feature may be important in the current discussion on the productivity gap e future research should be addressed to this issue.

6 Sensitivity Analysis

6.1 Sensitivity to policy parameters

We explore here the sensitivity of our generated series to changes in the values of model's parameters. In particular we study the effect of a change of the following policy parameters with respect to the baseline parameterization reported in Table 1: the taxation parameters (τ^Y and τ^I respectively income and corporate tax rate), the probability of being discovered (p) and the

surcharge factor on tax rate if discovered (s). The theory behind our model obviously predicts the sign of these changes. Accordingly, we can think of the analysis we perform here as a test of the “theory used for measurement”²⁰ predictions.

We analyze how the generated series changes in terms of max and min value, range and volatility, and we especially focus on the sign of these changes more than of their magnitude. The results for the series of underground employment are reported in Table 2 .

Table 3 Sensitivity of the generated underground employment series to policy shifts.

Baseline parametrization			0,050	0.13	0.39	0.26
Par	Baseline	New Value	Std Dev	Max	Min	Range
p	0.03	0.1	0.047	0.12	0.37	0.25
		0.01	0.051	0.13	0.39	0.26
s	1.3	2	0.049	0.13	0.38	0.25
		0.5	0.051	0.13	0.39	0.26
τ^Y	0.3436	0.4	0.056	0.15	0.44	0.29
		0.2	0.038	0.1	0.29	0.19
τ^Π	0.4155	0.5	0.061	0.16	0.47	0.31
		0.25	0.035	0.09	0.28	0.19

One major result of this exercise is that all the changes are economically meaningful. In general, an increase in the taxation parameters and a decrease in the evasion model’s parameters (probability of being discovered and surcharge factor) lead to a higher percentage of underground employment and, therefore, to an increase in the underground economy. The mechanism operates in the opposite way if we change the direction of the shifts. For facilitating the reading of Table 2 we can imagine that when income tax rate (τ^Y) moves from the baseline parameterization value (0.3436) to the new higher value (0.4 in the experiment), both the *min* and the *max* of the generated underground employment series increase (the baseline values are, in fact, respectively: *min*=0.13 *max*=0.39;

²⁰ With this term we refer to the methodology behind our generated series, discussed in section 4. In brief, here we are using at the same time theory and data: we generated our series with the optimality conditions from the model and with the observed data for the regular sector. The sensitivity analysis gains some robustness in this environment, for example it can offer more insights with respect to an analysis conducted with simulated data.

whereas after the policy shift these values are: $min=0.15$ $max=0.44$). Accordingly, the series shifts upward and then the generated underground employment quota predicted is higher.

Notice also that taxation and “enforcement” parameters show some differences in their impact on the underground series. The main pattern is that the generated series is more sensitive to shifts in the taxation parameters. An increase in both the income and corporate tax rates shifts the series up, widens the range the series spans and finally makes the volatility increase. On the other hand, a decrease of the enforcement parameters (s and p) does not have almost any effect on the series (anyway there is a slight effect on the volatility), while a increase of p and s brings about a reduction, though slight, in the underground sector and a lower volatility.

The sensitivity analysis is a successful feature of the model approach. With this approach each result is fully informative and we may consider the unsuccessful effects to improve the model and the estimation of non-observed variables. The fragility of existing specification about, say, the unique productivity shock, the different technologies, and so on, may be easily assessed and emended.

7. Concluding Remarks

A sizeable size of the GDP in all the industrialized countries is not reported. Prominent efforts to measure underground production and labor are, at their best, arbitrary. To limiting arbitrariness in measuring the underground sector we suggest a further method based on theoretical models that could be useful for defining benchmarks to compare empirical estimates.

Our expressions for labor and underground production are derived from the dynamic general equilibrium model. This produces two consequences.

First, our measurement of the underground economy considers both the intertemporal and intratemporal aspects.

Second, it is structurally and dynamically consistent with the behavior of the realizations of the model variables (regular and underground consumption, investment and capital stock, prices and factor productivity).

Unlike the standard econometric analyses, this approach is based on a global view and a global hold over system dynamics: we use it to infer the size of the unobserved underground sectors which is consistent with the optimal behavior of the representative households and firms. The models should be specified with the avowed purpose of studying the relationship between the regular and underground sector that characterize the economy under specific conditions, and the generated data solutions should be appreciated for their results not available by any other approach.

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Appendix

A.1 The Data.

Bovi's estimation (1999), is achieved using the currency demand approach. Specifically, the author utilizes an extension of Tanzi's monetary method (1980; 1983). This approach assumes that irregular transactions are undertaken in the form of cash payments. An increase in irregular economy implies an increase in the demand for liquidity. To estimate the excess of currency demand, the equation regressors for currency are chosen so as to include not only standard causes of the money demand (interest rates, habits payments and so on) but also the major factors that may generate the irregular economy phenomenon. Among these, tax and social security burdens are the most important causes. Zizza's estimates (2002) used in this work are generated from an application of Tanzi's monetary approach. Moreover, the equation regressors, used by Zizza to allow for a better controlling of the illegal economy, delimitate more precisely the size of the irregular sector. These methods calculate the excess of money setting to zero (or at their minimum level) the "underground factors" which should generate the underground behavior of firms and households.

Chiarini and Marzano's series (2004), as well, are generated using a modified version of the Tanzi's approach, in which the characteristics of non stationarity of the basic series are exploited through cointegration techniques to generate time series of the Italian size of the underground economy (1975-2001). The authors use different index for the tax burden, also controlling for some index of labor market disequilibrium (unemployment, mismatch in different branch of the labor market, social benefits for people who are only temporary out of job).

The National Statistic Institute (Istat) is starting to calculate the size of the underground production.

However, at moment, it is not available a quarterly time series but only annual data for few years (1992 to 2000). These estimations are based on a complex procedure, accepted at international level by Eurostat and OECD, consisting of three steps: first, a comparison between labor demand (firms) and labor supply (households) statistics to assess the size of unregistered work; second, a correction of the under-reporting of income by the enterprises; finally, a check of consistency of the economic aggregates through the balancing of the resources for each industry.

A.2 Business cycle facts and simulations.

In Table 1.A below we present the relative volatilities with respect aggregate output whereas Table 2.A includes the actual and simulated contemporaneous correlations with aggregate GDP. The baseline model predict absolute output volatility equal to 1.54 (0.05), which is close the actual one which is 1.44.

Table 1.A: VOLATILITIES		
	<i>Actual Data</i>	<i>Simulated Data</i>
C	0.62	0.47
W	0.83	1.06
W _M	-	0.83
W _U	-	1.81
N	0.54	0.54
N _M (*)	0.28	0.85
N _U (*)	0.46	0.92
Y	1.00	1.00
Y _M (*)	1.18	.16
Y _U (*)	2.21	1.23
I	5.25	9.15

The table includes the relative standard deviations with respect aggregate GDP standard deviations; (*) annual data. All statistics are computed based on 1000 simulations of 176 period lengths. Source: Actual Data, Schlitzer (1994); statistics for N_M, and N_U are based on Authors' calculation.

Table 2.A: CORRELATIONS		
	<i>Actual Data</i>	<i>Simulated Data</i>
C	0.85	0.91
N	0.52	0.60
I	0.78	0.90
W	0.69	0.55

Actual and simulated contemporaneous correlations with aggregate GDP. All statistics are computed based on 1000 simulations of 176 period lengths. Source: Actual Data, Schlitzer (1994).

The models' performance in predicting labor market dynamics (relative volatilities and contemporaneous correlations) deserve a special mention. Standard benchmark models (e.g. Hansen, 1985) have difficulty in replicating those figures, while our model is quite successful. This is indeed a welcome result, since it suggests the introduction of an underground sector improves ability of this class of models along labor market dimensions.

A.3 Comparison between underground economy and home production.

It is interesting to compare the theoretical structure of our model with household (home) production models.²¹ We focus on three selected issues: the commodities' number and their substitutability, the financing of capital investment and the insurance opportunities offered by home production and underground activities. First, consider the number of consumption goods and their substitutability. In the home production class of models there exist two goods, denoted as regular and underground commodities, each of which is produced with a sector specific technology. In addition, the preference specification allows for different degrees of substitutability between regular and underground goods. In contrast, in the model with an underground sector there exists only one homogenous good, which, however, is produced using two different technologies: one associated with the regular sector, and the other with the underground sector. In this environment it is natural to focus on the case of perfect substitutability between regularly-produced final output and underground-produced output.

The second difference concerns the financing of investments. In home production models, only regularly-produced goods can be consumed and invested, either into regular capital or into underground capital. There are no uses for home production output other than consumption - it cannot be sold or transformed into capital, for example, the way that regularly-produced output can. In the underground economy model, however, there exists only one capital stock (invested in the regular sector), but regularly-produced and underground-produced output can be transformed into regular capital without any adjustment cost. The underground sector offers an additional channel for financing capital stock accumulation, and an additional dimension along which firms can employ the available labor supply. Summarizing, while the home production model is a legitimate two sector model, our model could be more appropriately defined as a two technology model, since the same good is produced using two different technologies.

In addition, an underground sector offers profit smoothing opportunities for firms and insurance opportunities for consumers. More precisely, firms can smooth their profits by a proper allocation of labor demand between the two sectors, on a period by period basis. In addition, consumers can smooth not only consumption, by substituting over time consumption and investments, but also income, by allocating their labor supply across sectors, on a period by period basis. This mechanism is absent in models with home production.

²¹ Home production has been part of the standard labor paradigm, but only recently has it been introduced into macro models. See Benhabib, Rogerson and Wright (1991) and Greenwood, Rogerson and Wright (1995) for a survey.

Figures

Figure 1: Series of Italian underground economy (percentage GDP)

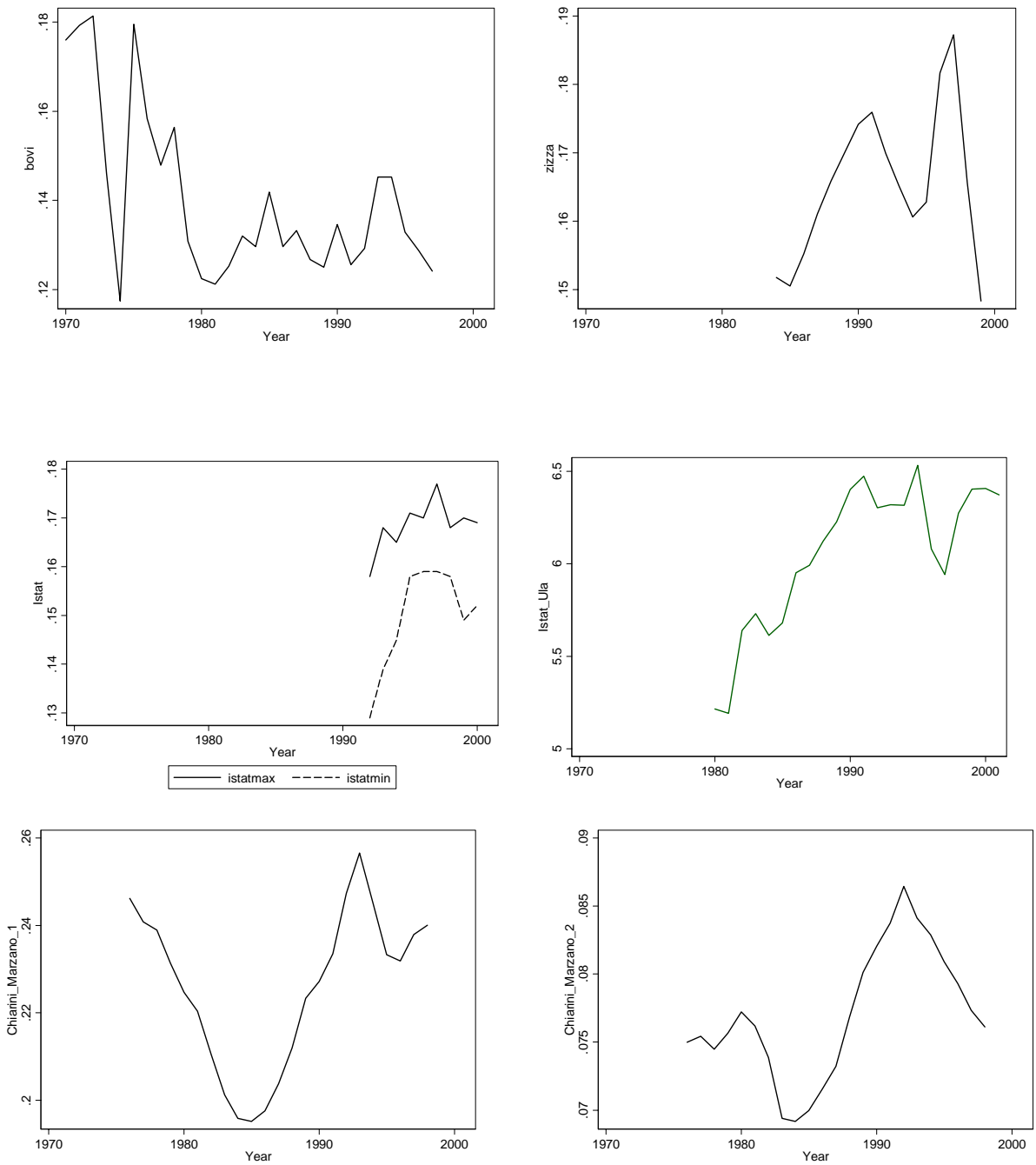


Figure 2 Share of employment in the underground sector

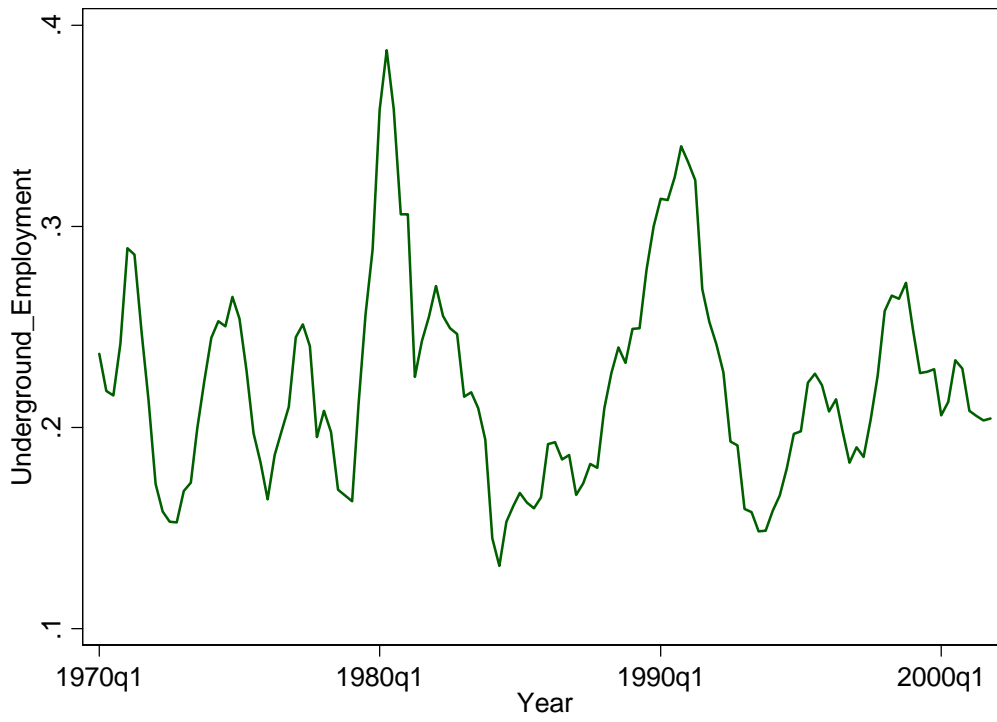


Figure 3 Share of underground GDP

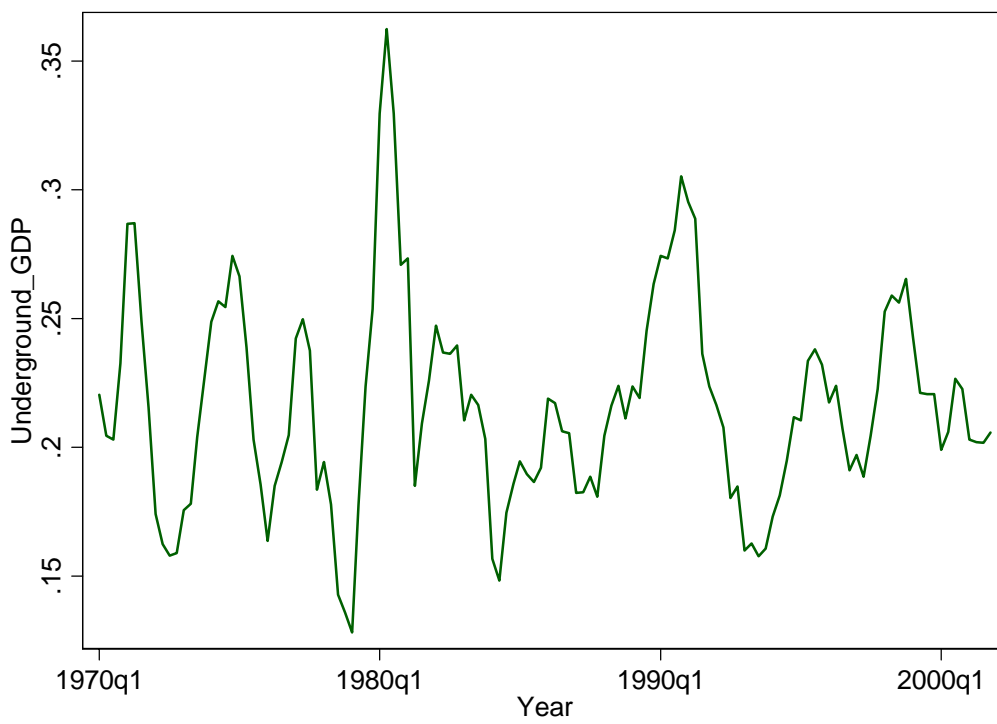


Figure 4: Underground employment share and filtered GDP

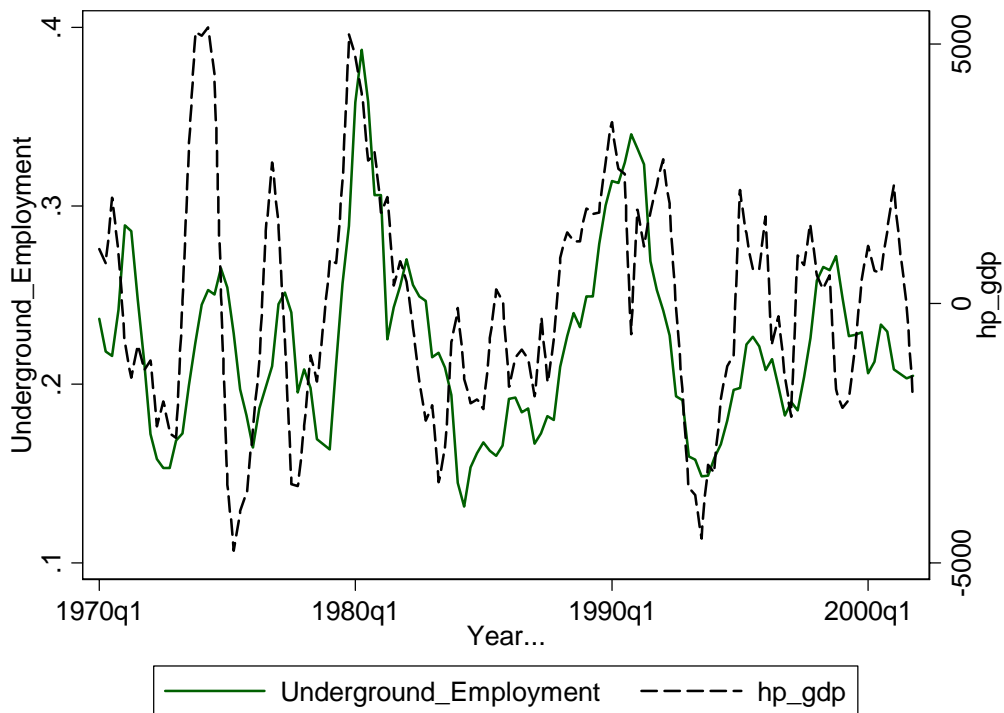


Figure 5: Productivity in the underground sector and GDP (both HP filtered)

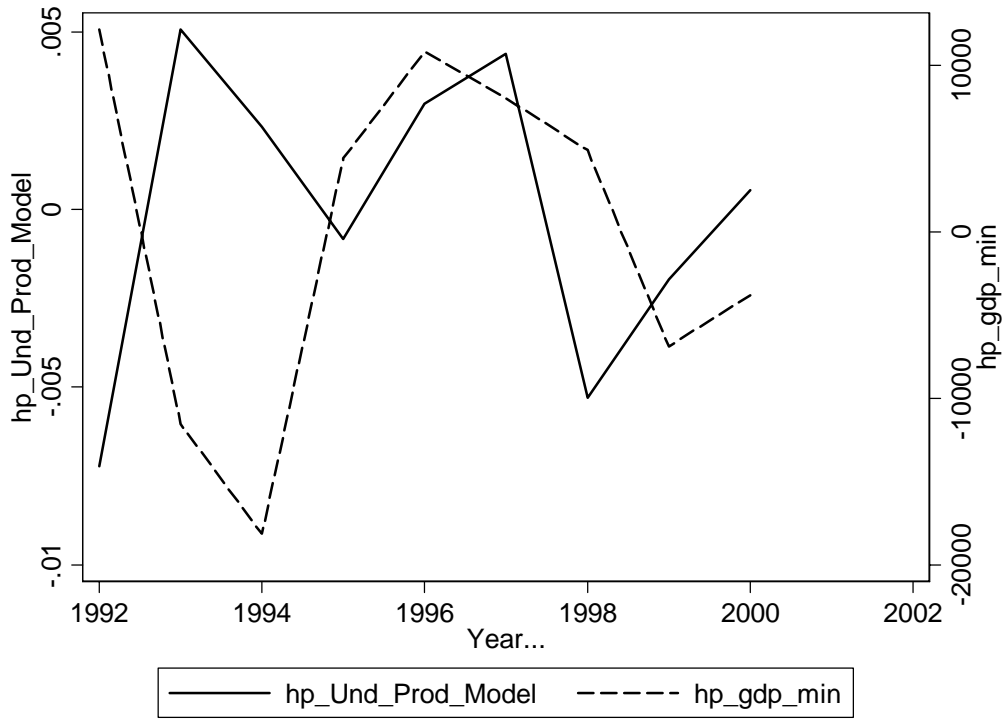


Figure 6: Productivity in the underground sector (HP filtered)

