

# Fiscal Policy under Indeterminacy and Tax Evasion.\*

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## Abstract

This paper shows under indeterminacy and tax evasion, an increase in corporate, labor or income tax rates pushes the economy into an expansionary pattern. These effects are reversed when the steady state is saddle-path stable.

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## 1 Introduction

The equilibrium effects and the role of fiscal policy in dynamic general equilibrium models of fluctuations have been object of thorough investigations in the last decade. Significant work has been done in fiscal policy analysis within neoclassical growth models.<sup>1</sup> Fiscal policy implications have been investigated also in the context of dynamic equilibrium models of fluctuations induced by indeterminacy in the equilibrium path, which is in turn due to increasing

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<sup>1</sup>Aschauer [2] and Baxter and King [5] are seminal contributions sharing an emphasis on the supply-side response of labor and capital to shifts in government demand and tax rates. Recent related contributions are: Braun [7], McGrattan [22], and Burnside Eichenbaum and Fisher [8].

return to scale at aggregate level. In this case, particular attention has been put on the impact of changes in the steady state levels of tax rates on the topological properties of the model's attractor (Guo and Lansing [19]; Christiano and Harrison [11]).<sup>2</sup>

The main finding of this paper is that recessionary fiscal policies (i.e. an increase in corporate, income or labor tax rates) induce expansionary effects for a class of dynamic general equilibrium models augmented with increasing returns to scale in production and tax evasion; these effects are reversed when the steady state is saddle path stable.

We analyze a one-sector dynamic general equilibrium model in which there are three agents: Firms, Households and a Government; furthermore, there is one homogeneous consumption good. The government levies proportional taxes on corporate revenues, labor and capital income flows, payroll taxes on labor services and balances its budget (in expected terms) for each period. Firms and households, being subject to distortionary taxation, use the underground labor market to evade taxes. Government faces tax evasion originating from the underground sector, and coordinates strategy to address abusive trust schemes. The model displays endogenous fluctuation due to increasing returns to scale and well behaved demand schedules for production inputs (in the sense that slope down).

Motivations for our analysis are theoretical and empirical.

As for the first one, the standard Farmer and Guo [15] models with indeterminacy and increasing returns to scale suffer from a number of undesirable features: e.g. a too high degree of increasing returns to scale in aggregate production, and an upward sloping labor demand schedule.<sup>3</sup> Busato et. al. [10] show how the introduction of an even tiny underground sector (justified by the incentive to evade distortionary taxation) can overcome some of the above mentioned undesirable features. Furthermore, this modification entails a theoretical mechanism allowing self-fulfilling prophecies to propel the business cycle, which is different from that proposed in the literature.

On the empirical side, underground activities are a fact in many countries, and there are

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<sup>2</sup>Devereux et al. [14] study the dynamic response to changes in government spending under increasing returns to scale, when the attractor is still a saddle point.

<sup>3</sup>This class of one sector models requires returns to scale greater than 1.6, while recent estimates suggest that US economy returns to scale are no larger than 1.2 (see, among the others, Basu and Fernald [4], Sbordone [24], Jimenez and Marchetti [20]).

significant indications that this phenomenon is large and increasing.<sup>4</sup> The estimated average size of the underground sector (as a percentage of total GDP) over 1996-97 in developing countries is 39 percent, in transition countries 23 percent, and in OECD countries about 17 percent (Schneider and Enste [25]). For the United States, the average size of underground activities ranges between 5 percent of GNP (in the Seventies: Tanzi [26]) and 9 percent of the GDP (in the Eighties, early Nineties: Paglin [23]; Schneider and Enste [25]).

The paper is organized as follows. Section 2 details the theoretical model, while Section 3 presents the topological properties of stationary state and discusses conditions for indeterminacy. Section 4 presents and discusses the model's response to fiscal policy shocks. Finally Section 5 concludes.

## 2 Structure of the model

### 2.1 Firms' sector

Production technology for the homogenous good  $y_{j,t}$  uses three inputs: physical capital, regular labor services, and underground labor services. The production function of firm  $j$  reads:

$$y_{j,t} = A_t k_{j,t}^\alpha (n_{j,t}^M)^{1-\alpha-\rho} (n_{j,t}^U)^\rho, \quad 0 < \alpha + \rho < 1, \quad (1)$$

where  $k_{j,t}$  denotes capital stock,  $n_{j,t}^M$  is regular labor,  $n_{j,t}^U$  represents irregular labor, and the quantity

$$A_t = \underbrace{\{K_t^\alpha (N_t^M)^{1-\alpha-\rho}\}^\eta}_{\text{“Regular” Ext.}} \underbrace{\{(N_t^U)^\rho\}^\zeta}_{\text{“Underground” Ext.}}, \quad (2)$$

represents an aggregate production externality: it passes through aggregate-average level of output ( $K$ ,  $N^M$  and  $N^U$  are the economy-wide levels of the three inputs) and has two different sources.

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<sup>4</sup>There is no universal agreement on what defines the underground economy, and obviously, the difficulty in defining the sector extends to the estimation of its size. There exist several synonyms for describing underground activities: underground activities, shadow or hidden economy. We are concerned with the size of the underground economy as encompassing activities which are otherwise legal but go unreported or unrecorded.

The quantity  $\{K_t^\alpha (N_t^M)^{1-\alpha-\rho}\}^\eta$  (the “regular” externality) is related to an external effect to that of standard one-sector models (e.g. Farmer and Guo [15]). The quantity  $\{(N_t^U)^\rho\}^\zeta$  (the “underground” externality) is specifically related to underground activities.<sup>5</sup> Externality parameter for regular labor  $\eta$  can be different from that of the underground one ( $\zeta$ ). This formulation adds generality to the analysis: when  $\eta = \zeta$  and there are neither tax evasion nor distortionary taxation, the model reduces to Farmer and Guo’s one.

As firms are homogeneous, overall level of output for a given (and equal for all firms) level of inputs utilization is given by:

$$Y_t = A_t \int_j \{k_{j,t}^\alpha (n_{j,t}^M)^{1-\alpha-\rho} (n_{j,t}^U)^\rho\} dj = K_t^{\alpha(1+\eta)} (N_t^M)^{(1-\alpha-\rho)(1+\eta)} (N_t^U)^{\rho(1+\zeta)}$$

Increasing returns to scale are a pure aggregate phenomenon (as equation (1) suggests), and returns to scale are constant at firm level, as each firm takes  $K$ ,  $N^M$  and  $N^U$  as given.

Firms evade taxes on total revenues and on labor services, by allocating labor demand to underground labor market. Firms, however, may be detected evading, with probability  $p \in (0, 1)$ , and forced to pay the statutory tax rates on revenues and the payroll tax rate on labor ( $\tau_t^R$  and  $\tau_t^N$  respectively), increased by a surcharge factor,  $s > 1$ , applied to the standard tax rate.<sup>6</sup>

When a firm is **not detected** evading (with probability  $1 - p$ ), its profit are denoted with  $\pi_{j,t}^{ND}$ . If **detected** evading (with probability  $p$ ), we denote firm’s profits as  $\pi_{j,t}^D$ ; both are defined below:

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<sup>5</sup>Underground labor services use the same capital stock that is used by regular labor. We could imagine that the same firm produces in the regular economy in the day, and in the underground economy by night, by using the same employees. This production scheme is denoted as “*moonlighting*” production. Bajada [3] loosely defines the underground economy to be all economic activity that contributes to value added and “goes unreported by a society’s measurement technique”. Bajada outlines several different activities that would lead to a distorted measure of national measurements: moonlighting is defined as failure to report income from a second job; or profit-businesses that are paid in cash and do not report this additional income i.e. hair dressers may report fewer clients than they really service; the failure to report interest earnings and barter. Cowell [13] offers additional details.

<sup>6</sup>The hypothesis that firms always evade is related to the use of underground labor. It must be noticed that such hypothesis is encapsulated in the definition of the production function: in order to have nonzero production,  $N_{j,t}^U$  must be positive in equilibrium.

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$$\begin{array}{lcl}
\pi_{j,t} & \rightarrow & \text{Detected} \\
& & \sim_p \\
& \searrow & \\
& & \text{Not Detected} \\
& & \sim_{(1-p)}
\end{array}
\quad
\begin{array}{l}
\pi_{j,t}^D = (1 - s\tau_t^\Pi)y_{t,j} - (1 + \tau_t^N)w_t^M n_{j,t}^M - (1 + s\tau_t^N)w_t^U n_{j,t}^U - r_t k_{j,t} \\
\pi_{j,t}^{ND} = y_{t,j} - (1 + \tau_t^N)w_t^M n_{j,t}^M - w_t^U n_{j,t}^U - r_t k_{j,t},
\end{array}$$


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where  $w_t^M$  is regular sector's wage,  $w_t^U$  is underground sector wage and  $r_t$  is capital remuneration rate. Tax rates can be hit by random shocks. Expected profit are computed by taking linear projection, i.e.  $\mathbb{E}\pi_{j,t} = (1 - p)\pi_{j,t}^{ND} + p\pi_{j,t}^D$ :

$$\mathbb{E}\pi_{j,t} = (1 - sp\tau_t^\Pi)y_{j,t} - (1 + \tau_t^N)w_t^M n_{j,t}^M - (1 + sp\tau_t^N)w_t^U n_{j,t}^U - r_t k_{j,t}.$$

The following condition, then, ensures a non-zero production:

**Condition 1 (No Bankruptcy)**  $(1 - sp\tau_t^\Pi) \geq 0$ .

For the rest of the analysis we will assume that condition 1 holds (in particular, for our parameterizations, described in section 3.3,  $1 - sp\tau_t^\Pi$  is positive). As markets are competitive, firm's behavior is described by the first order conditions for the (expected) profit maximization, with respect to  $k_{j,t}$ ,  $n_{j,t}^M$  and  $n_{j,t}^U$ :

$$\begin{aligned}
\alpha(1 - sp\tau_t^\Pi)\frac{y_{j,t}}{k_{j,t}} &= r_t \\
(1 - \alpha - \rho)(1 - sp\tau_t^\Pi)\frac{y_{j,t}}{n_{j,t}^M} &= (1 + \tau_t^N)w_t^M \\
\rho(1 - sp\tau_t^\Pi)\frac{y_{j,t}}{n_{j,t}^U} &= (1 + sp\tau_t^N)w_t^U.
\end{aligned} \tag{3}$$

Concavity of the production function (recall that firms take  $A_t$  as a constant) ensures the existence of a unique solution.

## 2.2 Households

Suppose that there exist a continuum of households, uniformly distributed over the unit interval. The  $j$ -th household's preference are represented by the following momentary utility

function:

$$u(c_{j,t}, n_{j,t}, n_{j,t}^U) = \ln c_{j,t} - B_0 \frac{(n_{j,t})^{1+\xi}}{1+\xi} - B_1 \frac{(n_{j,t}^U)^{1+\psi}}{1+\psi},$$

where  $c_{j,t}$  denotes household's consumption flow,  $n_{j,t}$  and  $n_{j,t}^U$  denote aggregate and underground labor supplies; the term  $B_0 \frac{(n_{j,t})^{1+\xi}}{1+\xi}$  with  $B_0 \geq 0$ , represents the overall disutility of working, while the last term,  $B_1 \frac{(n_{j,t}^U)^{1+\psi}}{1+\psi}$  with  $B_1 \geq 0$ , reflects the idiosyncratic cost of working in the underground sector<sup>7</sup>. Finally, the parameters  $\xi$  and  $\psi$  represent the inverse labor supply elasticities of aggregate and underground labor supplies, respectively.<sup>8</sup>

Aggregate labor supply equals sum of regular and underground labor flows:

$$n_{j,t} = n_{j,t}^M + n_{j,t}^U. \quad (4)$$

The household evade income taxes by reallocating labor services from regular to underground sector. Underground-produced income flows  $(w_t^U n_{j,t}^U)$  are, therefore, not subject to distortionary income tax rate  $\tau_t^Y$ , as the feasibility constraint below suggests:

$$c_{j,t} + i_{j,t} = (1 - \tau_t^Y) (w_t^M n_{j,t}^M + r_t k_{j,t}) + w_t^U n_{j,t}^U. \quad (5)$$

Capital stock is accumulated according to a customary state equation, i.e.

$$k_{j,t+1} = (1 - \delta)k_{j,t} + i_{j,t}, \quad (6)$$

where  $i_{j,t}$  represents net investments, and  $\delta$  denotes a quarterly capital stock depreciation rate.

Imposing a constant subjective discount rate  $0 < \beta < 1$ , and defining  $\mu_{j,t}$  as the costate variable, the Lagrangian of the household control problem reads:

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<sup>7</sup>Specifically, this cost may be associated with the lack of any social and health insurance in the underground sector.

<sup>8</sup>Assuming that  $\psi \geq \xi$  implies that the inequality is reversed when looking at the labor supply elasticities, i.e.  $\psi^{-1} \leq \xi^{-1}$ . It must be noticed that a reasonable hypothesis would rather be:  $\xi \geq \psi$ , so that overall labor supply elasticity is smaller than that of the underground-specific one. Furthermore, in our parameterizations (see section 3.3) we use  $\xi = \psi = 0$ , but results for the analysis of impulse response functions would be substantially analogous when allowing for a small difference  $\xi \gtrless \psi$  above the common zero value. Finally, recall that  $\xi$  relates to *total* labor supply, regular  $n_{j,t}^M$  plus underground one  $n_{j,t}^U$ .

$$\begin{aligned} \mathcal{L}_{0,j} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{j,t}, n_{j,t}, n_{j,t}^U) + \mathbb{E}_0 \sum_{t=0}^{\infty} \mu_{j,t} [(1 - \tau_t^Y) (w_t^M n_{j,t}^M + r_t k_{j,t}) + w_t^U n_{j,t}^U - c_{j,t} + \\ & -k_{j,t+1} + (1 - \delta)k_{j,t}], \end{aligned}$$

and the first order conditions obtain:

$$\beta^t c_{j,t}^{-1} = \mu_{j,t} \quad (7)$$

$$\beta^t B_0(n_{j,t}^M + n_{j,t}^U)^\xi = \mu_{j,t} (1 - \tau_t^Y) w_t^M \quad (8)$$

$$\beta^t B_0(n_{j,t}^M + n_{j,t}^U)^\xi + \beta^t B_1(n_{j,t}^U)^\psi = \mu_{j,t} w_t^U \quad (9)$$

$$E_t \{ \mu_{j,t+1} [(1 - \delta) + (1 - \tau_{t+1}^Y) r_{t+1}] \} = \mu_{j,t} \quad (10)$$

$$\lim_{T \rightarrow \infty} E_T \mu_{j,T} k_{j,T} = 0. \quad (11)$$

### 2.3 Government

The government budget, balanced in each period, is given by:

$$\tau_t^Y (w_t^M N_t^M + r_t K_t) + sp\tau_t^\Pi Y_t + sp\tau_t^N w_t^U N_t^U + \tau_t^N w_t^M N_t^M = \mathbb{E}_t REV_t = G_t, \quad (12)$$

where  $\mathbb{E}_t REV_t$  denote expected government revenues, which are allocated to government expenditure  $G_t$ . Notice that government balances its budget in expected terms since tax revenues collected from the underground sector depend on the probability of being detected  $p$ . Government expenditure is assumed to be wasteful, and the fiscal authority collects taxes in corporate sector after that production takes place.

Tax shocks - which are a source of intrinsic uncertainty - are defined by the following set of stochastic difference equations:

$$\widehat{\tau}_{t+1} = Q\widehat{\tau}_t + e_t, \quad (13)$$

where  $\widehat{\tau}_t = [\widehat{\tau}_t^Y, \widehat{\tau}_t^N, \widehat{\tau}_t^\Pi]'$ .  $Q$  is a matrix with elements  $[\varphi_{ii}^V]$ ,  $i = 1, 2, 3$ ;  $V = \{Y, N, \Pi\}$  on the

principal diagonal and zeroes elsewhere;  $e_t$  is a  $3 \times 1$  vector of i.i.d. random shocks and the covariance matrix  $\Sigma$  is a diagonal matrix  $[\sigma(e)_{ii}^V]$ ,  $i = 1, 2, 3$ ;  $V = \{Y, N, \Pi\}$ .

### 3 Model's Topological Properties

#### 3.1 Stationary State

Proposition 1 shows that the model has a unique stationary state for capital stock, and unique values for equilibrium regular and underground labor services. The stationary state quantities are derived under perfectly elastic labor supply schedules ( $\xi = \psi = 0$ ).<sup>9</sup>

**Proposition 1** *For  $\xi = \psi = 0$  there exists a unique stationary capital stock  $\bar{K} > 0$ , a unique stationary equilibrium for regular labor supply  $\bar{N}_M > 0$ , and unique stationary ratio  $\frac{\bar{N}_U}{\bar{N}_M}$  such that:*

$$\begin{aligned}\bar{K} &\simeq \left( \frac{(1 - \bar{\tau}^Y)(1 - sp\bar{\tau}^\Pi)\alpha}{\beta^{-1} - 1 + \delta} \right)^{\frac{1}{1-\alpha(1+\eta)}} \left( \frac{1 + \bar{\tau}_N}{1 + sp\bar{\tau}_N} \frac{B_0\rho(1 - \alpha - \rho)^{-1}}{(B_0 + B_1)(1 - \bar{\tau}^Y)} \right)^{\frac{\rho(1+\zeta)}{1-\alpha(1+\eta)}} (\bar{N}_M)^{\frac{(1-\alpha)(1+\eta)}{1-\alpha(1+\eta)}}; \\ \bar{N}_M &\simeq \frac{(1 - \alpha - \rho)}{B_0 \left( (1 - \bar{\tau}^Y) \left( \bar{\Psi}_M + \frac{\beta^{-1}-1+\delta}{1-\bar{\tau}^Y} \right) + \bar{\Psi}_U - \delta \right)} \left( \frac{\beta^{-1} - 1 + \delta}{\alpha(1 + \bar{\tau}^N)} \right); \\ \frac{\bar{N}_U}{\bar{N}_M} &= \frac{B_0\rho(1 - \alpha - \rho)^{-1}}{(B_0 + B_1)(1 - \bar{\tau}^Y)} \frac{1 + \bar{\tau}^N}{1 + sp\bar{\tau}^N},\end{aligned}$$

where  $\frac{(1-\alpha-\rho)}{(1+\bar{\tau}^N)} \frac{\beta^{-1}-1+\delta}{(1-\bar{\tau}^Y)\alpha} = \bar{\Psi}_M$  and  $\frac{\rho}{(1+sp\bar{\tau}^N)} \frac{\beta^{-1}-1+\delta}{(1-\bar{\tau}^Y)\alpha} = \bar{\Psi}_U$  and  $\bar{\tau}$  indicates the stationary level of a tax rate.

**Proof.** see Appendix. ■

The stationary equilibrium value for capital stock is derived under an approximation, which is necessary in order to obtain a closed form for  $\bar{K}$ ; it can be show, however, that results would not be significantly different if we derived its value numerically.

<sup>9</sup>This is a customary assumption, commonly accepted in this literature; see among the many Farmer and Guo [15]. It also allows to find a closed form for the stationary state. Indeterminacy arises, however, for  $\xi$ ,  $\psi \neq 0$ , as well (see footnote 8).

### 3.2 Conditions for Indeterminacy

For our parameterization (see Section 3.3 below), the model's attractor is a sink: the eigenvalues characterizing the stability properties of the unique stationary state are equal to  $0.8211 \pm 0.2123i$ . The topological properties of the model's attractor depend upon some crucial parameters; we restrict our attention to those characterizing the labor inputs' heterogeneity: particularly on  $\eta$  and  $\zeta$  and the tax rates. It can be shown that the model can display indeterminacy only if the following conditions are satisfied:<sup>10</sup>

**Condition 2 (NC)** (*necessary*):  $\beta < (1+\eta) \left\{ \left( 1 - (1 - \alpha - \rho) \frac{\bar{\tau}^N}{1+\bar{\tau}^N} \right) + \rho \left( \frac{1}{(1-\bar{\tau}^Y)(1+sp\bar{\tau}^N)} - 1 \right) \right\}$

**Condition 3 (SC)** (*sufficient*):  $\max \left\{ \frac{1}{\beta(1-\delta)}, \frac{\bar{\mathcal{R}}}{\bar{\mathcal{R}}-1} \right\} < \varepsilon_{\hat{N}^U}^{L^D} + \varepsilon_{\hat{N}^M}^{L^D} < \frac{\bar{\mathcal{R}}}{\bar{\mathcal{R}}-1}$ .

Condition **SC** is expressed in terms of the cross-elasticities of the (inverse) inputs demand functions; from equations (3) we have:  $\varepsilon_{\hat{N}^U}^{L^D} = \frac{\partial \hat{w}_t^M}{\partial \hat{N}_t^U} = \frac{\partial \hat{r}_t}{\partial \hat{N}_t^U} = (1 + \zeta) \rho$ ,  $\varepsilon_{\hat{N}^M}^{L^D} = \frac{\partial \hat{w}_t^U}{\partial \hat{N}_t^M} = \frac{\partial \hat{r}_t}{\partial \hat{N}_t^M} = (1 + \eta) (1 - \alpha - \rho)$ ; variables with the hat denote percentage deviations from the stationary state. The quantities  $\bar{\mathcal{R}}$  and  $\underline{\mathcal{R}}$  read:

$$\begin{aligned} \bar{\mathcal{R}} &= \frac{\delta(M - s_I) [1 - \beta(1 - \delta)] (1 - \alpha(1 + \eta)) + 2 [\delta\alpha(1 + \eta)M + s_I(2 - \delta)]}{\delta(M - s_I) [1 - \beta(1 - \delta)] (1 - \alpha(1 + \eta)) + 2 [\delta\alpha(1 + \eta)M + s_I(1 - \beta(1 - \delta))]} > 1, \\ \underline{\mathcal{R}} &= \frac{\delta s_I - \delta\alpha(1 + \eta)M}{s_I [1 - \beta(1 - \delta)] - \delta\alpha(1 + \eta)M} > 1, \end{aligned}$$

while  $M = 1 - \bar{G}/\bar{Y} = s_I + s_C$ ;  $s_I = \bar{I}/\bar{Y}$ ;  $s_C = \bar{C}/\bar{Y}$ .

The **Condition NC** suggests that in order to have indeterminacy it is necessary that the term  $\frac{s_I + s_C}{1 - \bar{\tau}^Y} = \left( 1 - (1 - \alpha - \rho) \frac{\bar{\tau}^N}{1 + \bar{\tau}^N} \right) + \rho \left( \frac{1}{(1 - \bar{\tau}^Y)(1 + sp\bar{\tau}^N)} - 1 \right)$ , augmented by the the “regular” externality parameter  $\eta$ , must be *large enough*; in other words, distortionary taxation should not drain too much resources away from the private sector, in order to allow the latter to form self-fulfilling beliefs.<sup>11</sup>

Taking  $\tau_Y$  as given, the higher the probability of being detected evading  $p$  (and/or the penalty surcharge factor  $s$ ), the more difficult it becomes to allocate resources to the underground sector, and the smaller the quantity  $\left( \frac{1}{(1 - \tau^Y)(1 + sp\bar{\tau}^N)} - 1 \right)$  gets. Consider the extreme

<sup>10</sup>See Busato et al. [10]

<sup>11</sup>Recall that government expenditure is wasteful in the model. It is difficult to figure out, off hand, whether our results still hold if government were rebating tax revenues either as consumption of private goods, or as investment to augment private inputs' productivity. This analysis is left for future investigation.

case where tax evaders are punished with an infinitely large penalty (that is  $s \rightarrow \infty$ ). The NC reads:  $\beta < (1+\eta) \left\{ \left( 1 - (1 - \alpha - \rho) \frac{\tau_N}{1+\tau_N} \right) - \rho \right\}$ , suggesting that the parameter region for indeterminacy shrinks when tax evasion becomes extremely costly, and it fails if labor taxes are too high ( $\tau_N > \frac{\beta - (1+\eta)(1-\rho)}{\alpha(1+\eta) - \beta}$ , for example).<sup>12</sup> Indeed, when tax evasion is extremely costly/risky, and when taxes are higher than a certain threshold, they “*tax away*” the externality, in the spirit of Guo and Lansing [19].

The picture is different if we take  $\tau_N$  as given. An increase in income tax rate  $\tau_Y$  monotonically increases the quantity  $\left( \frac{1}{(1-\tau_Y)(1+sp\tau_N)} - 1 \right)$ , easing, by this hand, the necessary condition. That happens because there is no probability of being detected evading income taxation (on the households’ side). Therefore, the higher the income tax rate, the higher would be the underground labor supply; in this sense, resources would be reallocated toward an input that ensures a tax-free externality. In this case we cannot claim that higher income tax rates tax away the externality.

The **Condition SC** is more enlightening about the nature of the economic process at basis of indeterminacy in our model. It suggests that the labor demand schedules should have a sufficiently large response to changes in equilibrium employment (i.e. the upper bound of SC), but, at the same time, that this response should not be too large (that is the Condition SC’s upper bound). In particular, the upper inequality in Condition SC can be rewritten in terms of elasticity of labor demand schedules to changes in capital stock ( $\varepsilon_{\hat{K}}^{L^D_M}$  and  $\varepsilon_{\hat{K}}^{L^D_U}$ ); it yields:

$$\varepsilon_{\hat{K}}^{L^D_U}, \varepsilon_{\hat{K}}^{L^D_M} > \frac{s_I}{s_I + s_C} \left\{ 1 + \frac{(1-\delta)(1-\beta)}{\delta} \left( \varepsilon_{\hat{N}^U}^{L^D_M} + \varepsilon_{\hat{N}^M}^{L^D_U} \right) \right\},$$

suggesting that regular and underground labor demands should react more strongly to changes in capital stocks rather than changes in labor services. In other words, the shifts of labor demands driven by changes in capital stock should be “larger” than those driven by changes in labor services. Both labor demand schedules are well behaved (in the sense that they slope down), compared to standard one-sector economy models where labor demand is upward

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<sup>12</sup>The NC condition fails when  $\beta > (1+\eta) \left\{ \left( 1 - (1 - \alpha - \rho) \frac{\tau_N}{1+\tau_N} \right) - \rho \right\}$ . This inequality can be recast in terms of  $\tau_N$  obtaining  $\tau_N > \frac{\beta - (1-\rho)(1+\eta)}{\alpha(1+\eta) - \beta}$ . Notice, moreover, that the quantity  $\frac{\beta - (1-\rho)(1+\eta)}{\alpha(1+\eta) - \beta}$  is quite small for reasonable parameters’ value.

sloping. Just observe that  $\frac{\partial \hat{w}_t^M}{\partial \hat{N}_t^M} = (1 + \eta)(1 - \alpha - \rho) - 1$  and  $\frac{\partial \hat{w}_t^U}{\partial \hat{N}_t^U} = (1 + \zeta)\rho - 1$  are both negative, for our parameterization.<sup>13</sup>

### 3.3 Parameterization

The model is parameterized for the U.S. economy; calibration is based on seasonally adjusted series from 1970 to 2001, expressed in constant 1995 prices. Actual data for the United States economy are drawn from Farmer [16]. The system of equations we use to compute the dynamic equilibria of the model depends on a set of 15 parameters, listed in Table 1 below; a starred parameter denotes the precise calibrated value.

Table 1: **Parameterization.**

Institutional Setting	$p^* = 0.057$	$s^* = 1.75$	
Disutility of Working	$B_0^* = 2.50$	$B_1^* = 2.00$	
Externality	$\eta^* = 0.45$	$\zeta^* = 0.20$	
Preferences	$\xi^* = 0.00$	$\psi^* = 0.00$	$\beta^* = 0.984$
Technology	$\alpha^* = 0.23$	$\rho^* = 0.088$	$\delta^* = 0.025$
Effective Tax Rates	$\bar{\tau}^N = 0.153$	$\bar{\tau}^Y = 0.1186$	$\bar{\tau}^\Pi = 0.355$

Notes: The parameters  $\xi^*$ ,  $\beta^*$ ,  $\psi^*$ ,  $\alpha^*$ ,  $\delta^*$  are from Farmer and Guo [15];  $B_0^*$  and  $B_1^*$  are set to match the labor share of regular and underground labor services  $\left(\frac{N_U}{N}\right)^* = 0.088$  and  $\left(\frac{N_M}{N}\right)^* = 0.912$ , following Schneider and Enste [25], where  $N^*$  is calibrated to 0.33.

The calibration of parameters that are more closely related to tax evasion and the underground economy deserve more attention.<sup>14</sup> For the *probability of being detected*  $p$ , we rely on Joulfaian and Ride [21], which estimate that the probability of auditing in the US ranges between 4.6% and 5.7%. We choose the higher value,  $p^* = 0.057$ , but results do not significantly

<sup>13</sup>This result is robust to a large set of parameters, as long as the regular externality  $\eta$  is sufficiently small. The quantity  $(1 - \alpha - \rho)$  is in fact positive and small. If  $\eta$  gets too big, the slope  $\frac{\partial \hat{w}_t^M}{\partial \hat{N}_t^M} = (1 + \eta)(1 - \alpha - \rho) - 1$  becomes positive. The same can happen to  $\frac{\partial \hat{w}_t^U}{\partial \hat{N}_t^U} = (1 + \zeta)\rho - 1$  when  $\zeta$  is high enough.

<sup>14</sup>Busato and Chiarini [9] calibrate a model for the Italian Economy, and Conesa, Diaz-Moreno and Galdon-Sanchez [12] calibrate a model for the Spanish economy.

change if we consider the lower value 4.7%.

To stop Abusive Trust Promoters, the Internal Revenue Service has recently undertaken a national coordinated strategy to address abusive trust schemes.<sup>15</sup> Violations of the Internal Revenue Code may result in civil penalties, which includes a fraud penalty up to 75% of the underpayment of tax attributable to the fraud in addition to the taxes owed. Therefore we set the *surcharge factor*  $s^* = 1.75$ .<sup>16</sup>

As for the average-long run levels of taxation the effective income tax rate  $\tau^Y$  and corporate tax rate  $\tau^\Pi$  are computed from the “Effective Tax Rates, 1979-1997”, Table H-1a, prepared by the Congressional Budget Office; social security tax rate is taken from [www.socialsecurity.com](http://www.socialsecurity.com); we choose the value applying for the 1990s and later, which equals  $\tau^N = 0.153$ .<sup>17</sup>

The externality parameters are set following Busato et al. [10]; aggregate level of returns to scale equals 1.42, which is lower than the original Farmer and Guo [15] calibration (1.61).

Shocks to tax rates are assumed to be permanent, that is the autocorrelation coefficients in (13) equal unity:  $\varphi_{11}^Y = \varphi_{22}^N = \varphi_{33}^\Pi = 1$ . It is therefore assumed that unexpected increases in tax rates is maintained in all subsequent periods, i.e. fiscal policy is based on a commitment to a non-transitory tax increase. Such hypothesis is not particularly restrictive, as the impulse response would maintain analogous qualitative features also in the case of purely temporary shocks ( $\varphi_{11}^Y = \varphi_{22}^N = \varphi_{33}^\Pi = 0$ ); the qualitative nature of the economy’s response only depends upon the topological properties of the attractor. In all exercises the size of each shock equals one unit standard deviation.

## 4 Fiscal Policy Implications

### 4.1 Impulse Response Functions

Figures 1, 2 and 3 include model’s response to a permanent increase in the three tax rates (specifically, after a positive one unit standard deviation innovation in  $\hat{\tau}^\Pi$ ,  $\hat{\tau}^Y$  and  $\hat{\tau}^N$ ). In

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<sup>15</sup>For more details about the Internal Revenue Service policy regarding abusive trusts, refer to Internal Revenue Service Public Announcement Notice 97-24, which warns taxpayers to avoid abusive trust schemes that advertise bogus tax benefits.

<sup>16</sup>Violations may also result in criminal prosecution; in this case there are penalties up to five years in prison for each offense.

<sup>17</sup>See <http://www.ssa.gov/OACT/ProgData/taxRates.html>

order to highlight the role of indeterminacy on the model's response to fiscal policy shocks, we confront impulse responses to increases in tax rates when the equilibrium is a sink (for  $\eta = 0.45$ ,  $\zeta = 0.2$ ) and when there is saddle path stability ( $\eta = \zeta = 0$ , but results holds also for nonzero values of  $\eta$  and  $\zeta$  guaranteeing saddle stability).

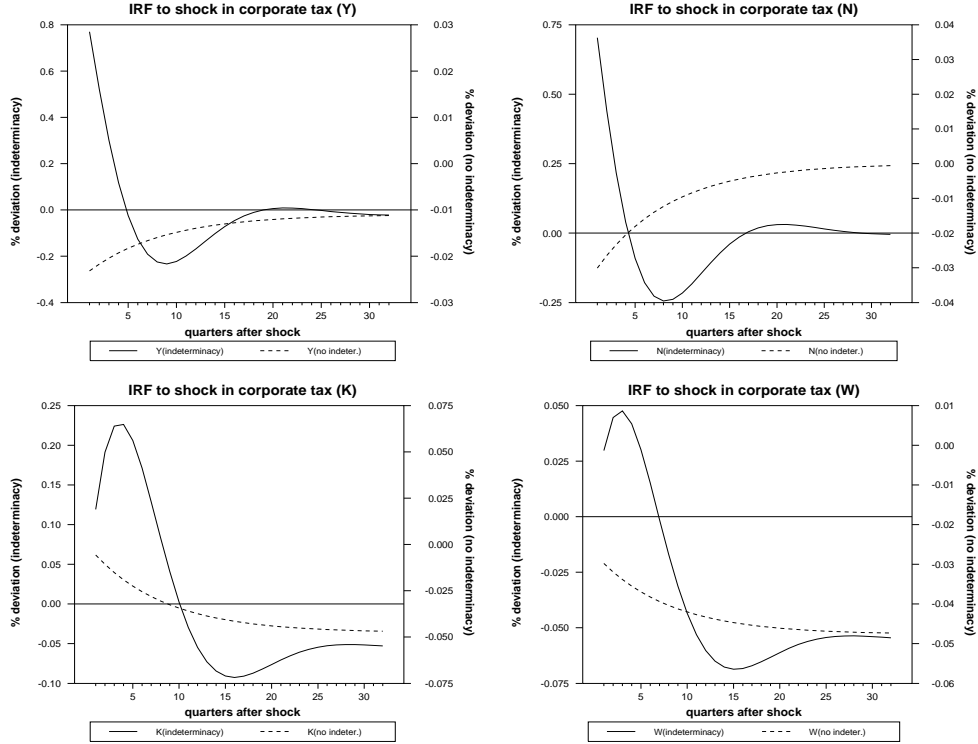


Figure 1: Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in corporate tax rate, with ( $\eta = 0.45$ ;  $\zeta = 0.2$ ) and without indeterminacy ( $\eta = \zeta = 0$ ). Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

Under indeterminacy an increase in tax rates pushes the economy into an expansion, while the economy plummets into recession when equilibrium is saddle-path stable. Figures 1, 2 and 3 show indeed that GDP, consumption, capital stock and aggregate equilibrium labor services increase; the smaller impact of  $\hat{\tau}^{\Pi}$  is related to the fact that in the model, the expected reduction in profits due to an increase in statutory corporate tax must be reduced by a factor  $sp$ . Under indeterminacy the Sufficient Condition SC ensures that an increase in input demands is capable to offset the recessionary impulse that would be generated by the increase in tax rates. On the contrary, when there are no aggregate externalities, this condition is not satisfied

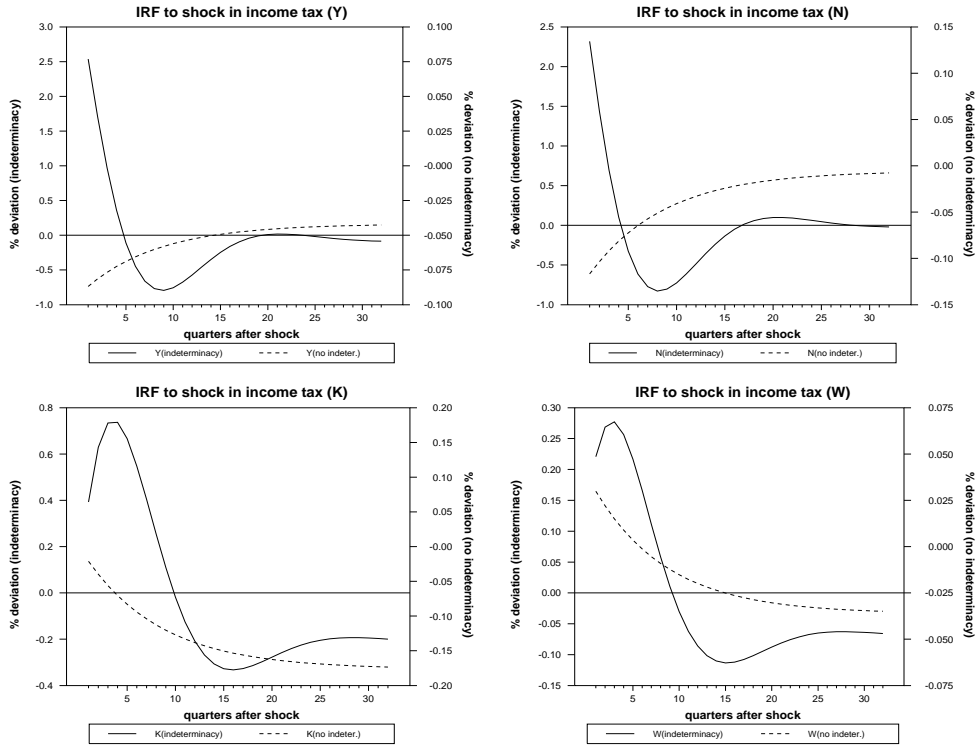


Figure 2: Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in income tax rate, with  $(\eta = 0.45; \zeta = 0.2)$  and without indeterminacy  $(\eta = \zeta = 0)$ . Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

any more, and a tax shock has just a recessionary effect.

An increase in one of the tax rates induces agents to allocate resources to the underground sector; agents in turn could forecast that such a reallocation may cause an economic expansion. But when there is just one transitional path to the stationary equilibrium, the reallocation to the underground sector is not capable to expand the economy and the “expansionary” prophecy would not be fulfilled. When instead there is indeterminacy, the expansionary prophecy can indeed be fulfilled; thus, the increase in tax rates can have an effect which is qualitatively similar to an expansionary sunspot shock.

The next section rationalizes these results by describing the theoretical mechanism operating in the model.

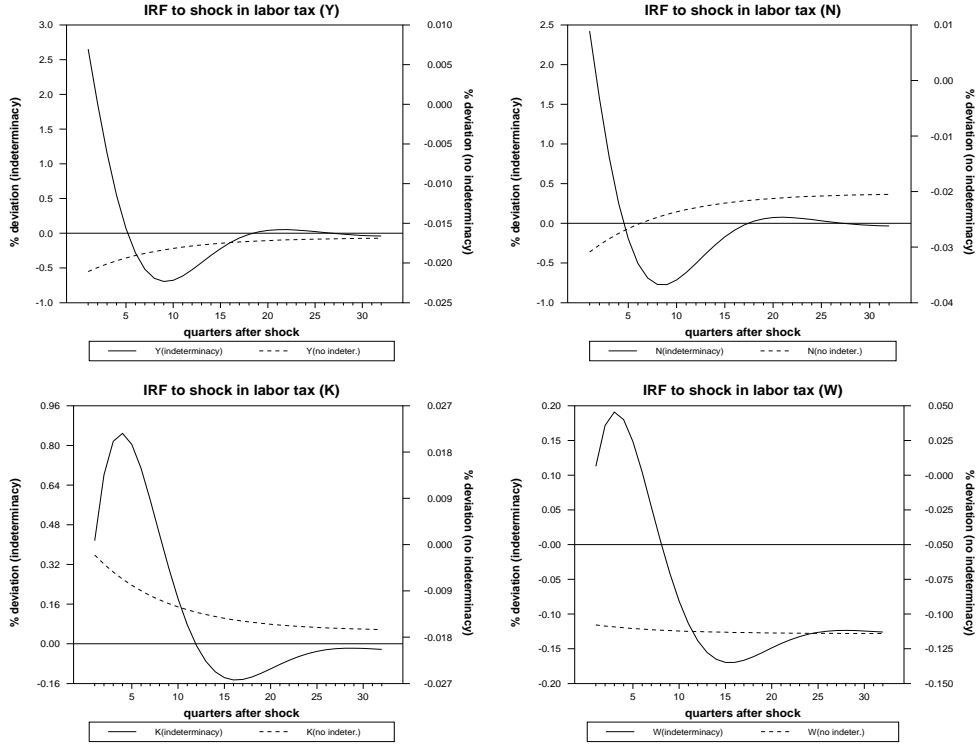


Figure 3: Predicted impulse response of output, capital stock, labor, and wage rate to a permanent increase in labor tax rate, with  $(\eta = 0.45; \zeta = 0.2)$  and without indeterminacy  $(\eta = \zeta = 0)$ . Solid Lines are responses under indeterminacy; dashed lines are responses when steady state is saddle-path stable.

## 4.2 Theoretical Mechanism

The model response is driven by a distinctive mechanism that operates through the labor market, and differs from that operating in standard one-sector models. We analyze qualitatively the behavior of labor markets after an unexpected and permanent increase in one of the tax rates.

Consider an exogenous increase in corporate tax rate (Figure 4 below). The regular ( $L_M^D$ ) and the underground ( $L_U^D$ ) labor demand schedules shift left after an increase in corporate tax rate ( $\uparrow \hat{\tau}^\Pi$ ) (along the corresponding supplies), reducing both  $\hat{N}_t^M$  and  $\hat{N}_t^U$ ; the labor demand cross-elasticity induces a further inward shift up to  $L^D(1)$ .<sup>18</sup> The fall in  $\hat{N}_t^M$  and  $\hat{N}_t^U$ , coupled with the increase  $\hat{\tau}^\Pi$ , reduces the interest rate  $\hat{r}_t$ ;<sup>19</sup> this produces two consequences. First,

<sup>18</sup>More formally, we have:  $\downarrow \hat{N}_t^M \Rightarrow \downarrow \hat{w}_t^U$  and  $\downarrow \hat{N}_t^U \Rightarrow \downarrow \hat{w}_t^M$ , because  $\frac{\partial \hat{w}_t^M}{\partial \hat{N}_t^U} > 0$  and  $\frac{\partial \hat{w}_t^U}{\partial \hat{N}_t^M} > 0$ .

<sup>19</sup>This can be seen from the linearized demand for capital:

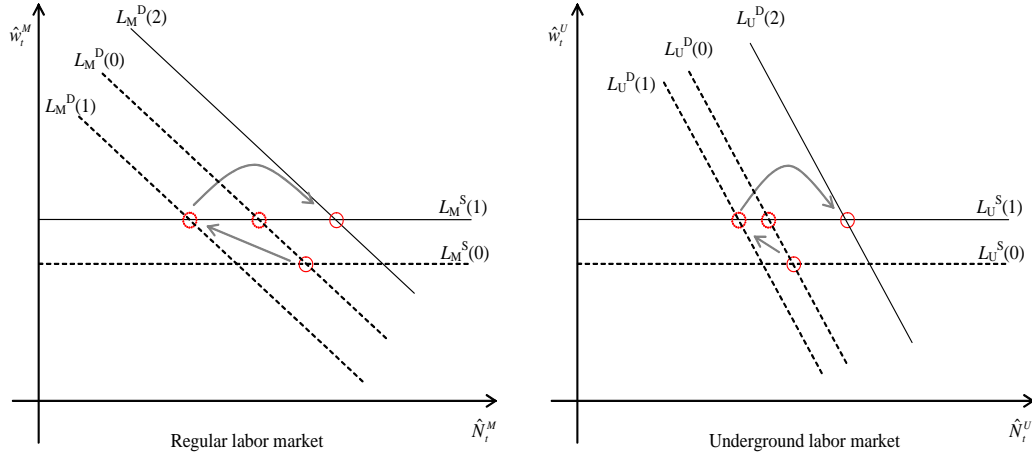


Figure 4: Labor Markets' response to a permanent increase in corporate tax rate. The left panel presents the regular labor market equilibrium; right panel includes the underground labor market equilibrium.  $(L_M^D), (L_M^S)$  : regular labor demand and supply;  $(L_U^D), (L_U^S)$  : underground labor demand and supply.

inputs' relative prices are now changed, as rental cost  $\hat{r}_t$  of capital is lowered with respect to the wage rates  $\hat{w}_t^M$  and  $\hat{w}_t^U$ ; thus labor is relatively more costly than capital. Second, the downward pressure on  $\hat{r}_t$  reduces the incentive to invest and makes consumption more attractive. The households could increase consumption, but they would need more resources to do so.

Here enters the crucial role played by tax evasion and the underground sector. In a perfect foresight equilibrium agents are aware that they can reduce their tax burden, increase their disposable income, and therefore afford a higher level of consumption, by allocating resources to the underground input. The resource reallocation toward the underground labor triggers an expansionary mechanism, because it increases returns to capital, regular labor services, and therefore equilibrium capital stock and equilibrium regular labor. The increase in the tax rate is an incentive to allocate more resources to the underground sector.

In addition, the sufficient condition for indeterminacy states that the elasticities of regular and underground labor demand schedule to capital stock should be sufficiently larger than a

$$\hat{r}_t = [(1 + \eta)\alpha - 1] \hat{K}_t + (1 + \eta)(1 - \alpha - \rho)\hat{N}_t^M + (1 + \zeta)\rho\hat{N}_t^U - \left[ \frac{sp\bar{\tau}^\Pi}{1 - sp\bar{\tau}^\Pi} \right] \hat{\tau}_t^\Pi$$

combination of elasticities to both kinds of labor services.<sup>20</sup> In this context the capital stock's impact is large enough that the labor demand schedules are shifted out up to  $L^D(2)$ ; the consequent increase in income induces a raise in equilibrium consumption, leading to a higher value function for households.

This depicts what we could name a “non-neoclassical effect” of fiscal policy in the sense that a rise in taxes has an expansionary effect over demand for good and services.<sup>21</sup>

An increase of income tax rate, as well as an increase in labor tax, still have an expansionary impact, as the operating mechanism is qualitatively similar to that previously presented. In summary, after a permanent increase in either corporate, income or labor tax rates the new equilibrium is characterized by a higher employment, higher wage rates, and higher interest rate on capital stock. This is a direct consequence of the self-fulfilling prophecies acting via capital accumulation and the cross elasticities of the three inputs demand schedules; a high-enough level of externality is needed for this kind of mechanism to be active.

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<sup>20</sup>To see this more clearly, consider the inverse (linearized) demand for regular labor (from equations 3):

$$\hat{w}_t^M = [(1 + \eta)\alpha] \hat{K}_t + [(1 + \eta)(1 - \alpha - \rho) - 1] \hat{N}_t^M + [(1 + \zeta)\rho] \hat{N}_t^U + \hat{P}_M$$

i.e. as a function  $\hat{w}_t^M = L_D^M\{\hat{N}_t^M; \hat{N}_t^U, \hat{K}_t, \hat{P}_M\}$ , where  $\hat{P}_M = \hat{P}_M(\hat{\tau}_t^I, \hat{\tau}_t^N)$  represents a set of parameters and other exogenous variables (mainly tax rates) and whose partial derivatives have the following signs:  $\frac{\partial L_D^M}{\partial \hat{N}_t^M} < 0$ ,  $\frac{\partial L_D^M}{\partial \hat{N}_t^U}, \frac{\partial L_D^M}{\partial \hat{K}_t} > 0$ . Symmetrically, the wage  $\hat{w}_t^U$  - the demand for underground labor - equals:

$$\hat{w}_t^U = [(1 + \eta)\alpha] \hat{K}_t + [(1 + \eta)(1 - \alpha - \rho)] \hat{N}_t^M + [(1 + \zeta)\rho - 1] \hat{N}_t^U + \hat{P}_U$$

and it is written as  $\hat{w}_t^U = L_D^U\{\hat{N}_t^M; \hat{N}_t^U, \hat{K}_t, \hat{P}_U\}$  where  $\frac{\partial L_D^U}{\partial \hat{N}_t^U} < 0$ ,  $\frac{\partial L_D^U}{\partial \hat{N}_t^M}, \frac{\partial L_D^U}{\partial \hat{K}_t} > 0$ . Now, the initial fall in each sector equilibrium labor services (that is a movement along each sector demand schedule) induces a further reduction in each sector employment through an inward shift of demand schedules (that is a schedule shift, induced by a change in the other-sector equilibrium employment).

<sup>21</sup>The short run reaction to fiscal policy discussed above is different from the so called “non-keynesian” effects, illustrated by several fiscal episodes mainly in Europe during the last two decades (Giavazzi and Pagano [17]). The literature on this topic has focused on the expansionary effects of a cut in public expenditures (Bertola and Drazen [6]) and the recessive effect of a reduction in tax rates (Giavazzi and Pagano [18]), both observed especially in nordic countries during the 80's–90's. It has linked these phenomena to the standard macroeconomic response to fiscal policy in neoclassical dynamic models; the non-keynesian reactions have been imputed to well known mechanisms, such as consumption smoothing and ricardian equivalence - when the issue of financing fiscal deficits is at stake. Our model abstracts from the issue of debt financing as budget is balanced in each period.

## 5 Conclusions

This paper studies fiscal policy in a one-sector dynamic general equilibrium model augmented with tax evasion and underground activities. The model displays increasing returns to scale due to externalities in regular and underground inputs, capable to induce sunspots and indeterminacy.

The main results depend on the combined presence of indeterminacy and tax evasion. We analyze the effect of shocks to taxation structure on the dynamic response of the economy, highlighting the differences with respect to the case of constant returns to scale. An increase in corporate, income or labor tax rates has an expansionary effect under local indeterminacy of the equilibrium path (i.e. when aggregate production technology displays increasing returns to scale high enough for producing indeterminacy). When instead there is saddle-path stability (returns to scale are low enough), the effects of tax shocks are customary, inducing a recession. This difference is due to a specific economic mechanism, which is distinctive of the indeterminacy case and acts through the cross elasticities of the inputs' demand functions.

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## Appendix

**Proof of Proposition 1.** Evaluating the FOCs (7)-(11) at the steady state yields:

$$B_0 = C^{-1} (1 - \tau^Y) w^M \quad (1)$$

$$C^{-1} w^U = B_0 + B_1 \quad (2)$$

$$r = \frac{\beta^{-1} - 1 + \delta}{1 - \tau^Y} \quad (3)$$

$$C = (1 - \tau^Y) (w^M N^M + rK) + w^U N^U - \delta K \quad (4)$$

$$r = \alpha(1 - \tilde{\tau}^\Pi) K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha-\rho)(1+\eta)} (N^U)^{\rho(1+\zeta)}$$

$$\tilde{w}^M = (1 + \tau^N) w^M = (1 - \alpha - \rho) (1 - \tilde{\tau}^\Pi) K^{\alpha(1+\eta)} (N^M)^{(1-\alpha-\rho)(1+\eta)-1} (N^U)^{\rho(1+\zeta)}$$

$$\tilde{w}_U = (1 + \tilde{\tau}^N) w^U = \rho(1 - \tilde{\tau}^\Pi) K^{\alpha(1+\eta)} (N^M)^{(1-\alpha-\rho)(1+\eta)} (N^U)^{\rho(1+\zeta)-1}$$

where  $\tilde{\tau}^N = sp\tau^N$  and  $\tilde{\tau}^\Pi = sp\tau^\Pi$ . Rewrite the Euler Equation (3) as

$$\begin{aligned} \frac{\beta^{-1} - 1 + \delta}{1 - \tau^Y} &= \alpha(1 - \tilde{\tau}^\Pi) K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha-\rho)(1+\eta)} (N^U)^{\rho(1+\zeta)-1} = \\ &= \alpha(1 - \tilde{\tau}^\Pi) K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha)(1+\eta)} \left( \frac{N^U}{N^M} \right)^{\rho(1+\zeta)} \end{aligned} \quad (5)$$

where the quantity  $\left( \frac{N^U}{N^M} \right)^\rho \frac{(N^U)^{\rho\zeta}}{(N^M)^{\rho\eta}}$  is approximated as  $\left( \frac{N^U}{N^M} \right)^{\rho(1+\zeta)}$ .

**Claim 1** Ratio  $\frac{N^U}{N^M}$  is stationary.

**Proof.** Combining (1) with (2), yields

$$\frac{B_0}{(B_0 + B_1)(1 - \tau^Y)} \left( \frac{\rho}{1 - \alpha - \rho} \right) \left( \frac{1 + \tau^N}{1 + \tilde{\tau}^N} \right) = \left( \frac{\bar{N}^U}{\bar{N}^M} \right) \quad (6)$$

Notice that the stationary labor ratio  $\frac{\bar{N}^U}{\bar{N}^M}$  is independent of any input-specific externality. ■

**Claim 2** The quantity  $K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha)(1+\eta)}$  is stationary

**Proof.** Combining (5) with (6), we obtain:

$$\begin{aligned} &\frac{\beta^{-1} - 1 + \delta}{\alpha(1 - \tau^Y)(1 - \tilde{\tau}^\Pi)} \left[ \frac{B_0}{(B_0 + B_1)(1 - \tau^Y)} \left( \frac{\rho}{1 - \alpha - \rho} \right) \left( \frac{1 + \tau^N}{1 + \tilde{\tau}^N} \right) \right]^{-\rho(1+\zeta)} \\ &= K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha)(1+\eta)} \end{aligned} \quad (7)$$

which establish our claim, since the left hand side is constant. ■

**Claim 3** *There exists a unique stationary equilibrium for aggregate labor supply  $\bar{N}^M > 0$ .*

**Proof.** *Now consider the feasibility constraint (4):*

$$C = K \left( (1 - \tau^Y) \left( \frac{w^M N^M}{K} + r \right) + \frac{w^U N^U}{K} - \delta \right) \quad (8)$$

and observe that the quantities  $\frac{w^M N^M}{K}$  and  $\frac{w^U N^U}{K}$  are stationary, too:

$$\frac{w^M N^M}{K} = \frac{(1 - \alpha - \rho) \beta^{-1} - 1 + \delta}{(1 + \tau^N) (1 - \tau^Y) \alpha} = \bar{\Psi}_M \quad (9)$$

$$\frac{w^U N^U}{K} = \frac{\rho}{(1 + \tilde{\tau}^N) (1 - \tau^Y) \alpha} = \bar{\Psi}_U \quad (10)$$

Substituting then (9), (10) into feasibility constraint (8), yields:

$$C = K \left( (1 - \tau^Y) \left( \bar{\Psi}_M + \frac{\beta^{-1} - 1 + \delta}{1 - \tau^Y} \right) + \bar{\Psi}_U - \delta \right) \quad (11)$$

To derive the stationary equilibrium for total labor supply, combine (1) with (11) into (1) to obtain:

$$\frac{B_0 \left( (1 - \tau^Y) \left( \bar{\Psi}_M + \frac{\beta^{-1} - 1 + \delta}{1 - \tau^Y} \right) + \bar{\Psi}_U - \delta \right) (1 + \tau^N)}{(1 - \tau^Y) (1 - \alpha - \rho) (1 - \tilde{\tau}^\Pi)} = K^{\alpha(1+\eta)-1} \frac{(N^M)^{(1-\alpha)(1+\eta)}}{N^M} \left( \frac{N^U}{N^M} \right)^{\rho(1+\zeta)}$$

Now, since  $K^{\alpha(1+\eta)-1} (N^M)^{(1-\alpha)(1+\eta)}$  and  $\frac{N^U}{N^M}$  are constant, the above equation gives the stationary value of  $\bar{N}^M$  in proposition 1. ■

The final step is to compute the stationary equilibrium value for  $\bar{K}$ . Combining (7) with the value of  $\bar{N}^M$  it turns out that  $\bar{K}$  is stationary, and it reads

$$\bar{K} \simeq \left( \frac{(1 - \tau^Y) (1 - \tilde{\tau}^\Pi) \alpha}{\beta^{-1} - 1 + \delta} \right)^{\frac{1}{1-\alpha(1+\eta)}} \left( \frac{1 + \tau^N}{1 + \tilde{\tau}^N} \frac{B_0}{(B_0 + B_1) (1 - \tau^Y)} \frac{\rho}{1 - \alpha - \rho} \right)^{\frac{\rho(1+\zeta)}{1-\alpha(1+\eta)}} (N^M)^{\frac{(1-\alpha)(1+\eta)}{1-\alpha(1+\eta)}}$$

. ■