

# Financial Structure, Informality and Development\*

Pablo N D'Erasmus

Hernan J Moscoso Boedo

Department of Economics

Department of Economics

University of Maryland

University of Virginia

January 29, 2009

Preliminary

## Abstract

The objective of the paper is to quantify the effects of observed entry, operating and exit frictions on the firm's production decisions and their aggregate effects across countries. We build a firm dynamics model with imperfect credit markets and endogenous formal and informal sectors. We find that if the average low income country was to adopt the financial, tax and formal sector structure of the US, their wage would increase by 30.46%. This increase in wages is not only a result of higher capital per worker but also one of higher total factor productivity. More specifically, TFP would increase by 18.73% and output per worker by 30.25%.

*Keywords:* Financial Structure, Informal Sector, Productivity, Policy Distortions.

*JEL Classifications:* D24, E26, L11, O16, O17

---

\*Correspondence: Pablo D'Erasmus, University of Maryland, Department of Economics, 3105 Tydings Hall, College Park, MD 20742, (301)405 3529, [derasmo@econ.umd.edu](mailto:derasmo@econ.umd.edu); Hernan Moscoso Boedo, University of Virginia, Department of Economics, PO Box 400182, Charlottesville, VA 22904, (434)924 7654, [hmoscoso@virginia.edu](mailto:hmoscoso@virginia.edu)

# 1 Introduction

In this paper we quantify aggregate effects across countries arising from institutional differences. In particular, institutional heterogeneity in terms of entry costs to the formal sector, differences in the tax structure (not only tax rates but also cost of tax compliance) and also in the efficiency debt enforcing mechanisms (measured as debt recovery rate and cost of the enforcing mechanism). The question we are after is how much of the international differences in total factor productivity, wages and output per worker are explained by differences in measured cost of doing business.

Firms take as given the different regulations and costs associated with them, and optimize accordingly. It should not be surprising to observe large number of firms producing in the underground economy in countries where the cost of entry and operating in the formal sector are extremely high and the benefits (the ability to enforce contracts) are almost negligible. Under these conditions firms endogenously choose to operate in the informal sector and are subject to restrictions as well. They do not pay taxes, but have a limited access to credit markets. Once the firm cannot borrow, the size and growth of the firm are limited. Also in the informal sector firms make explicit efforts to remain small in order to avoid detection by the authorities. That is, production ends up taking place at an inefficient scale and therefore output is below the optimal levels.

Informal activity is a feature in every country around the world. Figure 1 plots estimates by Djankov et. al. (2002) of the informal activity in the economy as a fraction of total GDP.<sup>1</sup>

---

<sup>1</sup>These estimates are mainly based on Schneider and Enste (2000), where they define the informal activities as “*legal value-added creating activities which are not taxed or registered and where the largest part can be classified as “black” or clandestine labor*”.

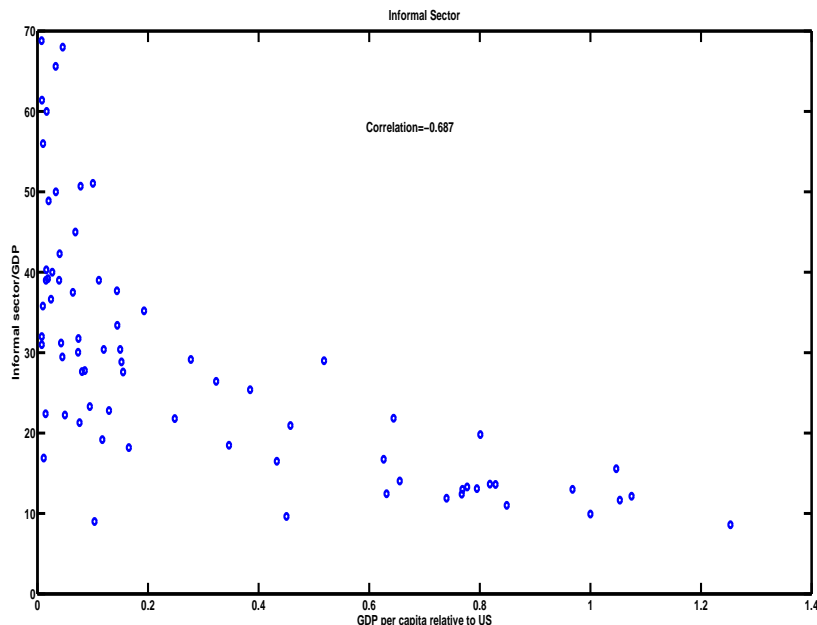


Figure 1: Size of the informal sector as reported by Djankov et al (2002)

Output produced outside the formal sector represents more than half of total output for Peru, Panama, Nigeria, Senegal, Madagascar, Bolivia, Egypt and Mozambique. On the other hand no country with GDP per capita above 80% of the US, has an informal sector larger than 20% of GDP.

We build a model of firm dynamics with endogenous entry and exit, that incorporates capital financing and bankruptcy decisions. The model allows for the existence of a formal and an informal sector. Entering and operating in the formal sector is costly but allows firms to access credit markets with a better commitment technology (given by the recovery rates and associated costs). Countries have access to the same production technologies but we impose country specific institutions measured by the World Bank as reported in its Doing Business database. The degree of debt enforcement varies across countries and affects the interest rate that firms face.

This is not the first paper in linking the costs of doing business to the aggregate level of output. In particular we trace our steps back to De Soto (2000) where he describes the process

by which a firm enters the formal sector in Peru. He argues that costly entry mechanisms to the formal sector prevent firms from producing at an efficient level. He measured the entry cost in time and resources and concludes that one of the reasons production is undertaken in the informal sector has to be the high costs associated with becoming formal. He continues to describe the functions of physical capital, and how it has a “parallel life” as collateral in the formal sector. Under this view, the benefit of formality lies in the ability to use physical capital as collateral to secure the interests of third parties in the events of contract breaches. On a similar approach the World Bank launched the “Doing Business ” project. Under this project, the costs associated with many dimensions of doing business are recorded across countries. They measure, among other things, costs to incorporate a firm, to obtain licenses to operate in a physical location, to hire workers, to pay taxes, and to close the business. The interesting feature of this project is that instead of collecting observed data for each of the aspect of doing business in a country (which depends on endogenous aspects such as the size of a firm), they run an experiment where they try to operate the same standardized firm across countries. This way the different costs across countries can be directly compared.

Our main experiment involves comparing an economy calibrated to the US and one where the main distortions are parameterized using data for countries across income groups. The results can be summarized as follows. If the average low income country was to adopt the financial, tax and formal sector structure of the US, their wage would increase by 30.46%. We find that this increase in wages is not only a result of higher capital per worker but also one of higher total factor productivity. TFP would increase by 18.73% and output per worker by 30.25%. Consistent with Figure 1 we obtain a negative correlation between income per-worker and the size of the informal sector. Moreover, the model endogenously generates plan-size distributions across countries that is in line with the findings of Alfaro et al. (2008). They show that the share of small firms in the formal sector decreases with total output.

Recent related literature on the distributional effects of frictions in a firms dynamics

context, include Hsieh and Klenow (2007) and Restuccia and Rogerson (2008). In both cases they back up the implied frictions in the firms environment necessary to generate the observed distribution of firms. In this paper, as in Moscoso Boedo and Mukoyama (2008), the frictions the firms face are those observed in the data collected by the World Bank. This paper introduces imperfect capital markets, and in that dimension, its closest papers include Antunes and Cavalcanti (2007), Castro, Clementi and MacDonald (2008), Erosa and Hidalgo Cabrillana (2008) and Quintin (2008). Castro et al. (2008) and Erosa and Hidalgo Cabrillana (2008) introduce asymmetric information in the financial markets. Antunes and Cavalcanti (2007) and Quintin (2008) study endogenous informal sectors that result from imperfect contract enforcement. This paper builds on this literature by analyzing a model of firm dynamics with uncertainty, both in the level and the volatility of the productivity of the firm. We also consider a different structure of the financial contracts where default costs are constrained by limited liability. Under limited liability tax enforcement is needed to generate endogenous entry to the formal sector.

The relevant empirical literature regarding firm dynamics across countries include Tybout (2000), Foster, Haltiwanger and Krizan (2001) and Alfaro, Charlton and Kanczuk (2007). Tybout (2000) is the only one that reports data on firms characteristics in the informal sector, while the other two use different data sources but focused on firms operating in the formal sector. Evidence from the informal sector is by definition hard to find. Firms operating in the informal sector make explicit efforts not to be detected and therefore, estimates of the size of the informal sector are very noisy. Schneider and Enste (2000) report various measures of the informal sector across countries and are the most comprehensive study to our knowledge regarding informality in a cross country setting.

The paper is organized as follows. In section 2 we present the institutional differences across countries as measured by the World Bank. We consider differences in the cost of entry to the formal sector, tax codes, and efficiency of the contract enforcement mechanisms. In section 3 we present the theoretical model, based on Hopenhayn and Rogerson (1993),

with physical and credit markets. Section 4 describes the stationary equilibrium of the model. Section 5 is devoted to the calibration of the model to the US data. In section 6 we experiment with different measured institutions and compute their impact in terms of total factor productivity and firms dynamics. Finally, section 7 concludes.

## 2 Institutional Differences across Countries

What firms have to do in order to start, operate and exit vary across countries. In order to compare these different costs the World Bank, through its Doing Business project follows a standardized firm across countries and measures regulations to entry, operate and exit. They measure the costs in terms of time and resources on many dimensions affecting the firm such as starting a business, getting construction permits, employing workers, getting credit, protecting investors, paying taxes, trading across borders, enforcing contracts and closing a business. Of particular interest to us in this paper are the cost to enter into the formal sector, the tax rate and tax compliance difficulty (while operating in the formal sector) and the efficiency of the debt enforcing mechanisms if the firm decides to default on its debt. These costs are depicted in Figure 7 against GNI per capita relative to the US.

The entry cost to the formal sector is constructed as in Moscoso Boedo and Mukoyama (2008). It has two parts. It includes the cost of incorporating a business and dealing with licences. Both costs have a monetary cost and a time cost (which is translated to monetary units by assuming that one worker has to be employed full time to go through the process of incorporation). The entry cost to the formal sector as a fraction of the wage (denoted by  $w\kappa$  from here on) varies greatly across countries, with high levels of  $\kappa$  observed only at the low end of the income distribution. For the US it is at 0.7% of GNI per capita while in Sierra Leone it is over 1000% of GNI per capita. In terms of time, in the US a business can start immediately while in Yemen and Syria it takes more than 5 years to start a formal business.

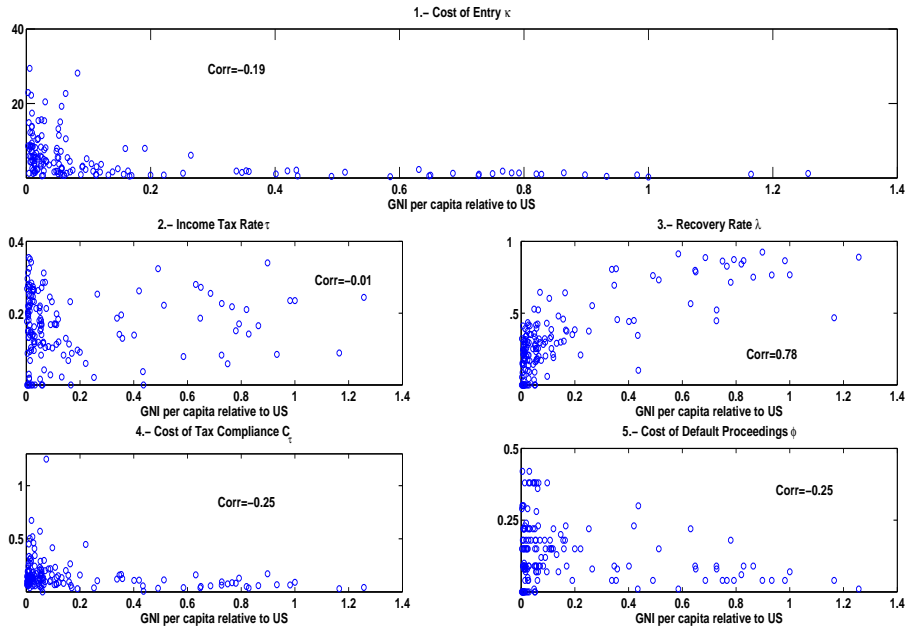


Figure 2: Cost to entry, income tax rate, cost of tax compliance, recovery rate and cost of default proceedings from the Doing Business Database. Outlyers ommited.

The income tax rate paid by the firms ( $\tau$  from here on) does not seem to have a pattern over the distribution of income per capita as shown by panel 2 in Figure 7. What does have a similar patter to the entry cost is the cost of tax compliance ( $wc_\tau$  from here on). This cost reflects the time it takes to pay taxes in each country. We assume that there is a full time worker during this time devoted to the tasks related with tax compliance and therefore translate time into costs as annual wages. The cost of paying taxes only displays levels above 10 weeks for countries below 20% the GNI per capita of the US. Paying taxes takes no time in the Maldives, 12 hours in the UAE, 187 hours in the US and above 1000 hours for Vietnam, Bolivia, Belarus, Cameroon and Brazil. This indicates a great deal of variation across countries in terms of the difficulty of their tax code. Firms have to bear not only the tax rate per se but also the cost of complying with the tax code, which at the low end of the income distribution does not seem to be insignificant.

Finally the efficiency system in the event of defaults has two components, a cost compo-

ment and a recovery rate. The cost of the system ( $\phi$  from here on), reported as a percentage of the estate's value, includes court fees, and the cost of insolvency practitioners such as lawyers fees and accountant fees. It goes from 1% of the estate's value in countries like Norway and Singapore to values above 40% in Sierra Leona, Liberia and the Ukraine and above 70% in the Central African Republic. The recovery rate that external lenders obtain once the firms decides to default on its debt ( $\lambda$  from here on). It is effectively zero for many extremely poor countries in sub-saharan Africa. On the other hand, it is only above 75% for developed countries. Note that it is the return obtained by the external creditor conditional on the borrower defaulting. It measures the cents on the dollar recovered from that point on, and includes different channels to solve the contract breach such as foreclosure, liquidation and reorganization, as reported by Djankov et al (2008).

### 3 Environment

The model is based on Hopenhayn and Rogerson (1993) adding capital and credit markets. Time is discrete. We set one period to be one year. There are three kinds of entities in the economy: establishments, lenders and consumers. The establishments produce the consumption and capital goods used in the economy. They are the capital owners and pay dividends to the consumers. The lenders lend to establishments. The consumers supply labor to the establishments and receive their profit.

#### 3.1 Consumers

There is an infinitely lived representative consumer who maximizes the expected utility:

$$\mathbf{U} = \mathbb{E}_s \left[ \sum_{t=s}^{\infty} \beta^{t-s} u(C_t) \right],$$

where  $E[\cdot]$  is the expectation operator,  $C_t$  is consumption and  $\beta \in (0, 1)$  is the discount factor. The household is endowed with one unit of labor that provides to the firm at the market wage rate  $w$  and receives the profits of the operating firms and a lump sum transfer from taxes collected on these firms. The consumer is also responsible for the total entry cost of new firms. All the saving and borrowing decisions are made by firms, so effectively the household is not allowed to borrow or save.

## 3.2 Technology

Next we describe the technology in this economy. The unit of production is an establishment. Each establishment is described by a production function  $f(z, k, n)$  that combines capital  $k$  and labor  $n$ . The parameter  $z$  represents the productivity of the plant. We assume that the technology has decreasing returns to scale. In particular, we let  $f(z, n, k) = zk^\alpha n^\gamma$  with  $0 < \alpha + \gamma < 1$  and  $\alpha, \gamma \in (0, 1)$ . The parameter  $z$  is iid across firms, distributed  $N(\mu_z, \sigma_j)$  where  $\sigma_j$  represents the volatility in industry  $j \in \{1, \dots, J\}$ .<sup>2</sup> Moreover,  $z$  follows a First Order Markov process with transition probability denoted by  $\eta_j(z'|z)$ .

We assume that there is a random fixed cost of production  $c_f$ , measured in units of output, that is iid across firms and over time with distribution  $\xi(c_f)$ . A firm that does not pay this fixed cost is not allowed to produce. Establishments own their capital and are allowed to borrow in the form of non-contingent debt  $b \geq 0$  from financial intermediaries. They finance investment either with debt or internal funds.

For a given production possibility, the firm can choose to produce in the formal sector or in the informal sector. If operates in the formal sector, the firm is subject to a proportional income tax  $\tau$  and a cost of filling those taxes  $c_\tau w$ . Creating a formal sector firm requires a cost  $\kappa w$  as reported by the Doing Business database. At the end of every period, firms operating in the informal sector can be detected by the government with probability  $\omega(k') = 1 - \exp\left[\frac{-k'}{v}\right]$ .

---

<sup>2</sup>We allow for variation in the volatility since there is evidence that there is substantial productivity heterogeneity even within industries. For example, the results in Syverson (2004) suggest that the interquartile range for measured revenue-based total factor productivity within narrowly defined sectors is around 30 log points.

Note that this probability is increasing on the size of the firm and the parameter  $v$  captures the degree of enforcement. Once informal sector firms are detected, they are forced to switch to the formal sector or to sell their capital and close.

Firms maximize expected discounted dividends  $d$ :

$$\mathbb{E}_s \left[ \sum_{t=s}^{\infty} \beta^{t-s} d_t \right],$$

where firms discount the future at the same rate of the representative consumer  $\beta$ .

Establishments can be created by paying a cost  $c_e$ . After paying this costs, firms observe their volatility  $\sigma_j$  and their initial level of productivity  $z_0$  that is draw from the distribution  $\nu(z_0)$ . Draws from this distribution are assumed to be i.i.d across firms. With this information in hand, they decide between staying out of the market and entering as a formal or informal firm.

### 3.3 Credit Markets

The credit industry makes loans to the formal and informal sector firms. Asset markets are not complete. In each period, firms borrow using only one period non-contingent debt denoted by  $b$ . Since there is perfect information, prices depend on firms characteristics given by their choice of sector (formal or informal), their future level of capital ( $k'$ ), their level of borrowing ( $b'$ ) and their current technology ( $z$ ). In particular, firms in the formal sector will borrow at price  $q^f(k', b', z)$  and firms in the informal sector will borrow at price  $q^i(k', b', z)$ . In each period, firms are allowed to default on its debt. A default triggers a bankruptcy procedure that liquidates the firm. When making a loan to a formal sector firm, lenders take into account that in the case of default they can recover up to a fraction  $\lambda$  of the original loan. The recovery rate of a loan to an informal sector firm that defaults is assumed to be zero. The formal bankruptcy procedure has an associated cost equal to a fraction  $\phi$  of the firm capital. The values of recovery rate  $\lambda$  and the bankruptcy cost  $\phi$  are obtained from the

Doing Business database.

Consistent with bankruptcy law across countries, we follow the limited liability doctrine. This limits the owner's liability to the firm's capital.

## 4 Equilibrium

We will focus on the stationary equilibrium of the model. In this equilibrium the wage rate and the schedule of prices are constant. Every equilibrium function depends on the set of loan prices and the wage rate. For ease of exposition we avoid making this dependence explicit. Before defining the equilibrium concept we study the problem of the agents in the economy. First, we describe the problem of incumbent establishments in the formal sector and informal sector respectively. Then, we describe the entrants' problem and the representative consumer problem. For a detailed explanation of the stationary distribution, aggregates and the definition of the equilibrium see to the appendix.

### 4.1 Formal Sector Incumbent

An incumbent establishment in the formal sector with productivity volatility  $\sigma_j$ , starts the period with capital  $k$ , debt  $b$  and previous productivity  $z_{-1}$ . Then, the establishment draws the fixed cost that is required for continuing the operation,  $c_f$ , and decides to operate the technology, exit after repayment of debts or default and liquidate the firm. If the establishment decides to exit after repayment, it receives  $k - b$ . If it decides to default and liquidate the firm, it receives the maximum between the remainder of the capital after paying the recovery rate to the outside investors net of default proceedings and zero. The value function of an establishment at this stage is denoted as  $W_j^f(z_{-1}, k, b, c_f)$ . If it decides to stay, it pays  $c_f$  and observes the current period's productivity  $z$ . The value function of operating a firm in the formal sector is denoted as  $V_j^f(z, k, b, c_f)$ . If the firm decides to operate, it decides the amount of employment in the current period,  $n$ , capital and assets for the following period,

$k'$  and  $b'$ , and produces. Recall that in the formal sector it is then subject to income taxes  $\tau$  and the fixed cost of preparing those taxes  $c_\tau$ .

The incumbent solves the Bellman equation

$$W_j^f(z_{-1}, k, b, c_f) = \max \left\{ \int V_j^f(z, k, b, c_f) d\eta_j(z|z_{-1}), \max\{0, (1 - \phi)k - \lambda b\}, k - b \right\}$$

and

$$V_j^f(z, k, b, c_f) = \max_{n, k', b'} d_j^f(z, k, b, c_f) + \beta \int W_j^f(z, k', b', c_f) d\xi(c_f)$$

$$\begin{aligned} d_j^f(z, k, b, c_f) &= (1 - \tau) [zk^\alpha n^\gamma - c_f - w(n + c_\tau)] \\ &\quad - k' + (1 - \delta)k + q_j^f(k', b', z)b' - b \geq 0 \end{aligned}$$

The solution to this problem provides the exit decision rule  $\chi_j^f(z_{-1}, k, b, c_f)$  that takes value 0 if the firm continues to operate, 1 if the firm decides to default and 2 if the firm decides to exit after repayment. We also obtain the optimal capital and debt decision rules  $k_j^{f'}(z, k, b, c_f)$  and  $b_j^{f'}(z, k, b, c_f)$  respectively for a formal producing firm.

## 4.2 Informal Sector Incumbent

Firms operating in the informal sector start the period having been detected by the government or not. They can be detected by the government with probability  $\omega(k')$ . Once detected, the choice for the establishment is to switch operations to the formal sector or pay operating fees and close. The value function for the detected informal incumbent given the fix operating cost  $c_f$  and volatility  $\sigma_j$  is given by

$$\widetilde{W}_j^i(z_{-1}, k, b, c_f) = \max \left\{ \int \widetilde{V}_j^f(z, k, b, c_f) d\eta_j(z|z_{-1}), \max\{k - w\kappa, 0\} \right\}$$

Where

$$\tilde{V}_j^f(z, k, b, c_f) = \max_{n, k', b'} \tilde{d}_j^f(z, k, b, c_f) + \beta \int W_j^f(z, k', b', c_f) d\xi(c_f)$$

$$\begin{aligned} \tilde{d}_j^f(z, k, b, c_f) &= (1 - \tau) [zk^\alpha n^\gamma - c_f - w(n + c_\tau + \kappa)] \\ &\quad - k' + (1 - \delta)k + q_j^f(k', b', z)b' - b \geq 0 \end{aligned}$$

The solution to this problem provides the exit decision rule  $\tilde{\chi}_j^i(z_{-1}, k, b, c_f)$  that takes value 0 if the firm continues to operate as formal and 1 if the firm decides to close and default.

On the other hand, undetected informal establishments, can continue operations in the informal sector. After observing the fix operating cost  $c_f$  informal establishments of volatility  $\sigma_j$  have the option of continuing informal operations, switching to the formal sector or default and exit.

The informal incumbent establishment solves the following Bellman equation

$$W_j^i(z_{-1}, k, b, c_f) = \max \left\{ \int V_j^i(z, k, b, c_f) d\eta_j(z|z_{-1}), \int \tilde{V}_j^f(z, k, b, c_f) d\eta_j(z|z_{-1}), k \right\}$$

where

$$V_j^i(z, k, b, c_f) = \max_{n, k', b'} d_j^i(z, k, b, c_f) + \beta \left[ \begin{array}{l} \omega(k') \int \tilde{W}_j^i(z, k', b', c_f) d\xi(c_f) \\ + [1 - \omega(k')] \int W_j^i(z, k', b', c_f) d\xi(c_f) \end{array} \right]$$

$$\begin{aligned} d_j^i(z, k, b, c_f) &= zk^\alpha n^\gamma - c_f - wn \\ &\quad - k' + (1 - \delta)k + q_j^i(k', b', z)b' - b \geq 0. \end{aligned}$$

The solution to this problem provides the exit decision rule  $\chi_j^i(z_{-1}, k, b, c_f)$  that takes value 0 if the firm continues to operate as informal, 1 if the firm decides to default and 2 if it decides to switch operations to the formal sector. We also obtain the optimal capital and debt decision rules  $k_j^i(z, k, b, c_f)$  and  $b_j^i(z, k, b, c_f)$  for an informal producing firm and capital and debt decision rules  $\tilde{k}_j^i(z, k, b, c_f)$  and  $\tilde{b}_j^i(z, k, b, c_f)$  for an informal that switches to formal.

### 4.3 Entrants

In order to draw from the pool of ideas, potential entrants pay  $c_e$ . The value of a potential entrant  $W_e$  is given by:

$$W_e = \int W_j^i(z_0, 0, 0, 0) d\mu(\sigma_j) d\nu(z_0) - c_e.$$

Effectively, an entrant is equivalent to an informal sector firm with no capital, no debt and where the cost of production  $c_f$  equals zero. After paying  $c_e$ , entrants observe volatility  $\sigma_j$ , the productivity level  $z_0$ , which affects the conditional distribution from which the first productivity parameter will be drawn. Then decide whether to enter the formal (by paying the additional entry cost) or the informal sector. The assumption that the potential entrant draws the volatility of his/her productivity process and the first realization of the process, is similar to what Castro, Clementi and MacDonald (2008) do. Differences in the volatility of the process together with differences in initial productivity are going to generate differences in the decisions by the entrants and by the potential lenders. That introduces differences in behavior as a function of volatility and contract enforceability. In this equilibrium,  $W_e = 0$  will hold.

The solution to this problem provides the entry decision rule  $\chi_j^i(z_0, 0, 0, 0)$  that follows the decision rule of the incumbent in the informal sector.

## 4.4 Lenders

Lenders make loans to formal and informal establishments taking prices as given. Profits for a loan  $b'$  to a formal firm with future capital  $k'$  and volatility  $\sigma_j$  is

$$\pi_j^f(k', b', z) = -q_j^f(k', b', z)b' + \frac{1 - p_j^f(k', b', z)}{1 + r}b' + \frac{p_j^f(k', b', z)}{1 + r} \min \{ \lambda b', (1 - \phi)k' \},$$

where  $p_j^f(k', b', z)$  denotes the default probability of this borrower.

Profits for a loan  $b'$  to an informal firm with future capital  $k'$  and volatility  $\sigma_j$  is

$$\begin{aligned} \pi_j^i(k', b', z) = & -q_j^i(k', b', z)b' + [1 - \omega(k')] \frac{[1 - p_j^i(k', b', z)]}{1 + r} b' \\ & + \omega(k') \frac{[1 - \tilde{p}_j^i(k', b', z)]}{1 + r} b', \end{aligned}$$

where  $p_j^i(k', b', z)$  denotes the default probability of the undetected borrower and  $\tilde{p}_j^i(k', b', z)$  the default probability of the detected borrower. In equilibrium, the schedule of prices will adjust so  $\pi_j^f(k', b', z) = 0$  and  $\pi_j^i(k', b', z) = 0$  for all  $(j, k', b', z)$ .

## 4.5 Consumer's Problem

Since we are looking for the stationary equilibrium, aggregates in the economy are constant. This, and the fact that the consumer supplies the unit of labor inelastically, implies that the consumer maximizes expected discounted utility subject to the following budget constraint:

$$C = w + \Pi + T - E + X,$$

where  $\Pi$  is the total profit,  $T$  is the lump-sum transfer from the income tax,  $E$  the aggregate entry cost and  $X$  the exit value of firms. Note that the consumer is not making any decision, only receiving transfers, profits and wages which are consumed period by period.

## 5 Calibration

In this section we calibrate the model to the US economy. The basis for this calibration can be found in Moscoso Boedo and Mukoyama (2008) and D’Erasmus (2007).

The productivity process is given by

$$\ln(z_{t+1}) = (1 - \rho)\mu_z + \rho \ln(z_t) + \epsilon_{t+1}$$

With  $\epsilon_{t+1} \sim N(0, (1 - \rho^2)\sigma_j^2)$ . Where  $\sigma_j^2$  is the idiosyncratic variance of the process obtained by the firm when it entered. Note that  $\sigma_j$  will be given by a grid of fixed values, which will remain constant for the firm as long as it keeps operating. We let  $\sigma_j \in \{\hat{\sigma}(1 - h), \hat{\sigma}, \hat{\sigma}(1 + h)\}$ , where  $\hat{\sigma} = 0.2305$  is calibrated to match the value in the U.S. manufacturing sector as estimated by Cooper and Haltiwanger (2006). To calibrate  $h$  we use the NBER data on productivity across industries also for the U.S. manufacturing sector.<sup>3</sup> We first obtained the distribution of productivity standard deviation across industries. The distribution displays a considerable mass around the median. Finally, we set  $h = 43.16\%$  to cover 90% of the observed volatilities when the set of volatilities is centered at the median. Accordingly, we set the entrant’s distribution over volatilities  $\mu(\sigma_j)$  to 1/3 for all  $\sigma_j$ . The autocorrelation parameter,  $\rho$ , which is constant across  $j$ , is set to .885 as estimated for the U.S. manufacturing sector by Cooper and Haltiwanger (2006). The process will be discretized to obtain the grid for  $z$  and the transition probabilities  $\eta_j(z'|z)$  following the method explained in Tauchen (1986). The number of grid points for  $z$  is set to 11. The value of  $\mu_z$  is chosen to match the average size of the formal operating establishment in the US. From the transition matrix  $\eta_j(z'|z)$  we can derive the unconditional probabilities  $\eta_j^*(z)$ . We set the distribution of initial shocks  $\nu_j(z_0) = \eta_j^*(z)$ .

The operating fixed cost can take values in  $\{0, \hat{c}_f, +\infty\}$ . We calibrate  $\hat{c}_f$  and the distribution of fixed operating costs  $\xi(c_f)$  to match the exit rates across the size distribution of firms

---

<sup>3</sup>The data is publicly available in <http://www.nber.org/nberces/nbprod96.htm> and contains data on productivity across industries in the manufacturing sector.

as in Moscoso Boedo and Mukoyama (2008). The labor share  $\gamma$  is set to 0.64 a standard value and the capital share is based on previous estimates of the degree of decreasing returns to scale at the firm level. In particular, we set  $\alpha = 0.21$ , so  $\alpha + \gamma = 0.85$  as reported in Restuccia and Rogerson (2008). The risk free interest rate  $r$  is set to 4% per year to match the average real return on a 5 year T-bill for the last 30 years. We assume that  $\beta = \frac{1}{1+r}$ . The depreciation rate  $\delta$  is set to 7%. The value of the entry cost  $c_e$  is calibrated as in Hopenhayn and Rogerson (1993). In particular, we normalize the wage rate to 1 and find the value of  $c_e$  that satisfies the free entry condition with equality. The parameter  $\nu$  controls the detection probability and the size of the informal sector establishment. It turns to be the case that most new establishments in the formal sector correspond to switchers from the informal sector. For this reason,  $\nu$  is chosen to match the average size of the opening establishment in the formal sector.

Finally, the parameters  $\{\tau, c_\tau, \kappa, \lambda, \phi\}$  vary across countries and are chosen to match the values reported in the Doing Business data set for the U.S. economy (see Table 4 below).

Table 1 displays the parameter values and a summary of the moments used for the calibration.

Table 1: Model Parameters

Parameter		Value	Moment (US economy)
Discount Factor	$\beta$	0.9615	Avg. yearly return 5-year T-Bill
Depreciation Rate	$\delta$	0.07	Manufacturing Sector
Labor Share	$\gamma$	0.64	Labor Share
Capital Share	$\alpha$	0.21	Degree of Decreasing Returns
Autocorrelation	$\rho$	0.885	Manufacturing Sector
Mean process	$\mu_z$	0.4055	Avg. Operating Establishment
Median Std Dev	$\hat{\sigma}$	0.2305	Manufacturing Sector
Std Dev grid	$h$	0.4316	NBER productivity distribution
Positive Operating Cost	$\hat{c}_f$	7.5	Exit Rate Distribution
Distribution Op. Costs	$\{\xi(\hat{c}_f), \xi(\infty)\}$	$\{.10, .042\}$	Exit Rate Distribution
Detection Prob. Parameter	$\nu$	10	Avg. Entrant Size

Table 2 shows moment values used for the calibration in the data and those produced by the model.

Table 2: Target Moments

Moment	US Data	Model
Average Formal Est	17.6	17.7
Average Entrant Est	8.3	9.06
Exit Rate Distribution		
by Employment Size	(%)	(%)
1-4	14.88	12.41
5-9	6.72	7.81
10-19	5.57	5.45
20-49	4.91	4.21
50-99	4.58	4.20
100-249	4.16	4.20
250-499	3.90	4.20
500-	4.22	-

After the calibration exercise is done, we test the model in different dimensions. In particular, we ask how the distribution of entrants and operating establishments generated by the model compare with those of the US. We also contrast the average entry and exit rate. The data regarding the distribution of establishments (in the formal sector) in the US comes from the Statistics of US Business (SUBS) dataset for the years 2003-2004. It is the same data used in Moscoso Boedo and Mukoyama (2008).<sup>4</sup> Table 3 shows that the model does a good job in matching the number of small establishments not only for the operating firms but also for newly created formal establishments. It is important to note that the entrant distribution in our model is an endogenous object and not the result of the calibration of the initial firm productivity. By construction the average entry rate and exit rate in the model

<sup>4</sup>A description of this data set can be found in <http://www.census.gov/epcd/subs/introusb.htm>. Statistics of U.S. Businesses basic data items are extracted from the Business Register, a file of all known single and multi-establishment employer companies maintained and updated by the U.S. Census Bureau. The annual Company Organization Survey provides individual establishment data for multiestablishment companies. Data for single-establishment companies are obtained from various Census Bureau programs, such as the Annual Survey of Manufactures and Current Business Surveys, as well as from administrative records of the Internal Revenue Service, the Social Security Administration, and the Bureau of Labor Statistics.

are identical. Compared to the US, the entry and exit rates are only 26% and 16% lower.

Table 3: Test of the Model

Employment	Operating Formal		Opening		Operating Informal Model(%)
	Model (%)	US Data (%)	Model (%)	US Data(%)	
1-4	39.2	48.5	49.1	72.0	87.5
5-9	25.8	21.5	24.0	14.0	9.0
10-19	15.7	14.2	15.3	7.3	2.8
20-49	10.7	9.8	9.2	4.3	0.7
50-99	5.2	3.3	1.9	1.4	0
100-249	2.8	1.9	0.5	0.7	0
250-499	0.6	0.5	0	0.1	0
500 +	0	0.3	0	0.1	0
Entry Rate	8.55	11.6			
Exit Rate	8.55	10.2			

## 6 Main Experiment

In this paper we ask whether institutional differences, quantified by differences in the entry cost to the formal sector, the tax structure and the efficiency of debt enforcing mechanisms can help explain aggregate differences across countries. In order to implement this experiment, we use the Doing Business database for the year 2009 to obtain  $(\lambda, \phi, \tau, c_\tau, \kappa)$  for each income group as reported by the World Bank. Table 4 shows parameter values for the US economy (used in the benchmark calibration) and those of High, Upper Middle, Lower Middle and Low Income countries.

We will compare the benchmark case (calibrated to the US) with the equilibrium across income groups. Our main experiment can be described as follows: Calibrate the model to the US economy by using  $(\lambda, \phi, \tau, c_\tau, \kappa)_{US}$ . This implies iterating on the set of prices  $q_j^f(k', b', z)$  and  $q_j^i(k', b', z)$  until lenders make zero profit on each contract and adjusting the mass of potential entrants until the labor market clears. Since for this case we nor-

Table 4: Frictions across income groups

	$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$
US	0.767	0.070	0.235	0.089	0.264
High (HIC)	0.653	0.089	0.166	0.076	1.303
Upper Middle (UMIC)	0.122	0.274	0.158	0.129	2.802
Lower Middle (LMIC)	0.166	0.249	0.176	0.193	6.631
Low (LIC)	0.146	0.141	0.214	0.151	27.892

malize  $w = 1$ , we obtain the value of  $c_e$  that satisfies the free entry condition. Then, for each income group, we adjust the group specific parameters to  $(\lambda, \phi, \tau, c_\tau, \kappa)_g$ , where  $g \in \{HIC, UMIC, LMIC, LIC\}$  and iterate on prices  $w$ ,  $q_j^f(k', b', z)$  and  $q_j^i(k', b', z)$  until lenders make zero profits and the free entry condition is satisfied (for given  $c_e$ ). Finally we adjust the mass of potential entrants until labor markets clear.

Tables 5 displays equilibrium moments for the US economy and for each income group.

Table 5: Model Moments Across Income Groups

Moment	US	HIC	UMIC	LMIC	LIC
Informal Sector % GDP	4.34	4.60	5.40	6.41	15.44
Informal Sector % Active Firms	23.41	31.02	38.11	47.95	85.56
Wage $w$	1.00	1.00	0.95	0.89	0.76
Relative TFP $\equiv (Y/K^\alpha)/(Y_{us}/K_{us}^\alpha)$	1.00	0.99	0.96	0.92	0.84
Relative Output per Worker	1.00	1.00	0.96	0.89	0.77
Entry/Exit Rate	8.55	7.98	7.17	5.79	4.52
Leverage Ratio Formal Sector	0.39	0.36	0.28	0.27	0.28
Leverage Ratio Informal Sector	0.99	0.72	0.56	0.51	0.53
Avg. Employment Formal Sector Est.	17.75	20.48	25.03	37.77	94.65
Avg. Employment Informal Sector Est.	2.63	2.17	2.31	2.81	2.91
Total Capital Stock Formal Sector	1.97	2.13	1.87	1.71	1.23
Total Capital Stock Informal Sector	0.04	0.03	0.03	0.03	0.05

The joint effect of the frictions in terms of the variables shown in table 5 is mostly monotonic in the income level. Therefore it is convenient to compare the US vs. the LIC.

The predicted wage for the LIC is 76.65% of the US. This means that if the LIC were to adopt the financial, tax and formal sector structure of the US, its wage would increase by 30.46%. This increase in wages is not only a result of higher capital per worker but also one of higher total factor productivity. TFP would increase by 18.73% and output per worker by 30.25% if the average LIC was to adopt the US measured institutions.

Consistent with Figure 1, the size of the informal sector (measured by % of output) triples compared to that of the US. The model also predicts a small average informal firm. In line with the predictions of our model, Schneider and Enste (2000) find evidence that poorer countries typically have a large informal sector populated by small production units.

Table 6: Model Distribution Establishments: US vs LIC

Employment	Operating Formal (%)		Operating Informal (%)		Opening Formal (%)		Exit Rates (%)	
	US	LIC	US	LIC	US	LIC		
1-4	39.2	9.2	87.5	83.5	49.1	0.0	12.4	6.6
5-9	25.8	15.3	9.0	12.7	24.0	0.1	7.8	4.5
10-19	15.7	15.3	2.8	2.6	15.3	3.4	5.5	4.4
20-49	10.7	22.1	0.7	1.0	9.2	39.3	4.2	4.4
50-99	5.2	13.8	0	0.1	1.9	29.3	4.2	4.2
100-249	2.8	13.9	0	0	0.5	24.5	4.2	4.2
250-499	0.6	6.1	0	0	0	3.1	4.2	4.2
500 +	0	4.2	0	0	0	0.4	-	4.2

The implied differences between the US and the average LIC in TFP are the result of capital misallocation. This can be seen, for example, by comparing differences in entry/exit rates, the size of the informal sector and the distribution of employment of operating and opening establishments to the formal sector as shown in table 6. These distributions are almost normal in the LIC (centered in the 20-49 workers bin), whereas they are monotonically decreasing in the US. Equilibrium debt contracts offered in LIC not only constraint the level of capital that firms can operate (aggregate capital decreases by 36%) but also creates incentives to allocate it to less productive firms. We will discuss the latter effect in more

detail in the following section when we separate the effect of changes in the financial structure from other changes.

Differences in  $\kappa$  and the financial structure endogenously determine what kind (in terms of volatility of productivity, and its level) of establishment chooses to enter and operate in the formal sector. The model captures the negative correlation between income per worker and plant size in the formal sector as described in Alfaro, et al (2008).<sup>5</sup> They find that the share of small firms is bigger in the US than in less developed countries.

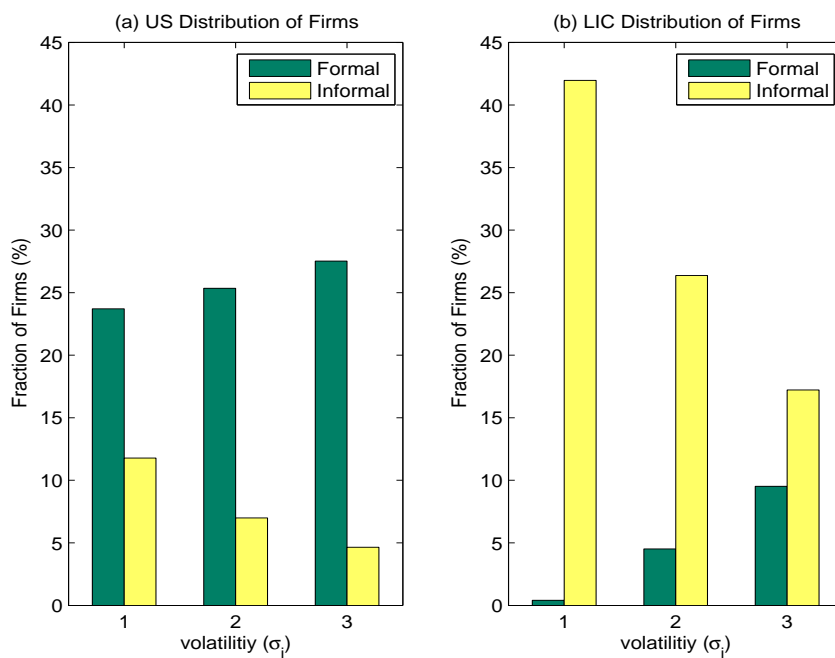


Figure 3: Distribution of Firms across Sectors: US vs LIC

Figures 3, 4 and 5 show that the volatility of productivity  $\sigma_j$  is critical in determining the entry decision to the formal sector. While the distribution of volatility in the formal sector of the US is almost uniform around 25%, in the average LIC, high volatility firms represent almost twice of those with moderate volatility and more than 10 times those with the lowest volatility. The ex-ante probability of each  $\sigma_j$  equals 1/3. However, we can

<sup>5</sup>They use plant-level data for 79 developed and developing countries from the Dun & Bradstreet's World-Base.

observe from Figure 3 that the total measure of firms across  $\sigma_j$  is  $(0.355, 0.323, 0.322)$  and  $(0.424, 0.309, 0.267)$  for the US and LIC respectively. This implies a substantial increase in the number of firms operating with  $\sigma_1$  when we move from the US to the LIC frictions.

Entry and exit rates are related to the capital misallocation. Low exit rates tend to be associated with economies where relatively low productive firms survive longer. In our experiment the entry/exit rate for the US is 8.54% and for the average LIC it is only 4.52%. Note that the probability of drawing the highest operating cost  $c_f = +\infty$  is calibrated to be 4.2%. This suggests that the establishments choosing to exit in the LIC are only those that obtain the worst  $c_f$  draw. On the contrary, in the US, the exiters are not only those with the worst draw of  $c_f$ , but also almost half of those drawing  $\hat{c}_f$ .

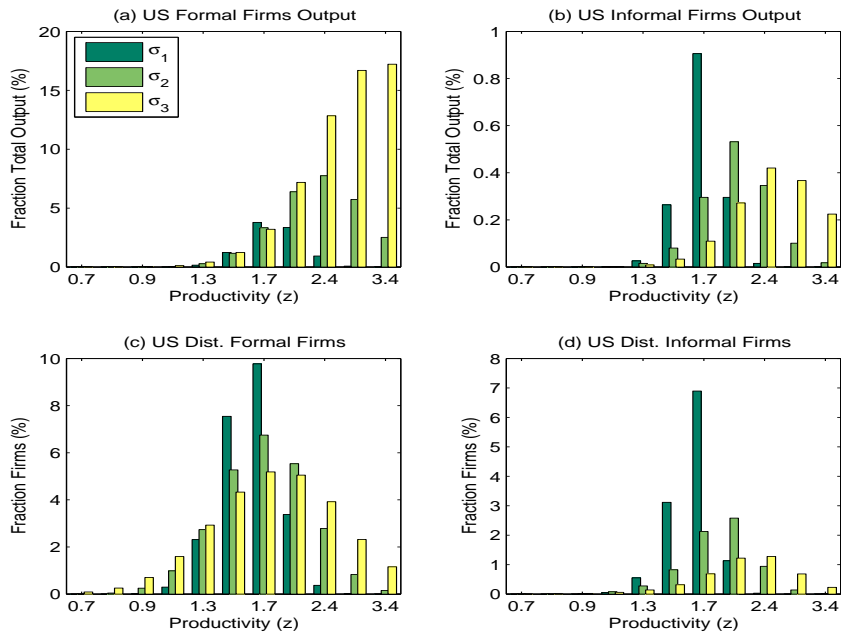


Figure 4: Dist. of Output and Firms across Sectors, Volatility and Productivity in the US

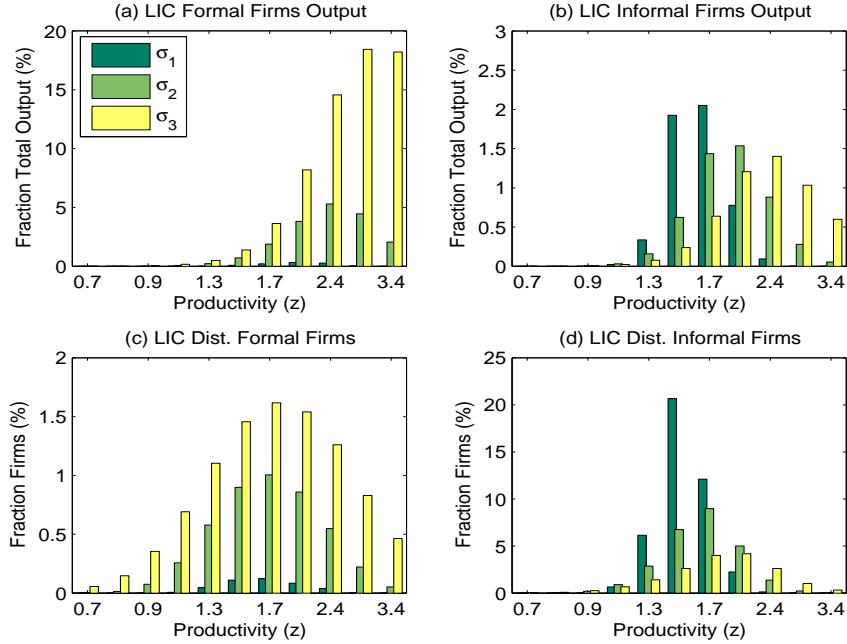


Figure 5: Dist. of Output and Firms across Sectors, Volatility and Productivity in LIC

The fact that we predict differences in the exit behavior of the establishments comes from observed differences in the financial contracts offered. Panels (a) and (b) in Figure 6 display these prices for the formal sector in the US and LIC respectively. In the LIC, there are effectively only two prices. Either risk free, corresponding to lighter colors, or extremely high interest rates (which implies a tight borrowing constraint given the low recovery rate). In the US, the relatively high recovery rate allows for lending at an intermediate price ( $q_j^f(k', b', z)$ ) that is higher than than of the LIC, allowing for the existence of establishments with pairs  $(k', b')$  that are not feasible in the LIC. The financial differences are also reflected in the equilibrium leverage ratio. On average, formal and informal firms display higher leverage ratios ( $b/k$ ) in the US than in the LIC.

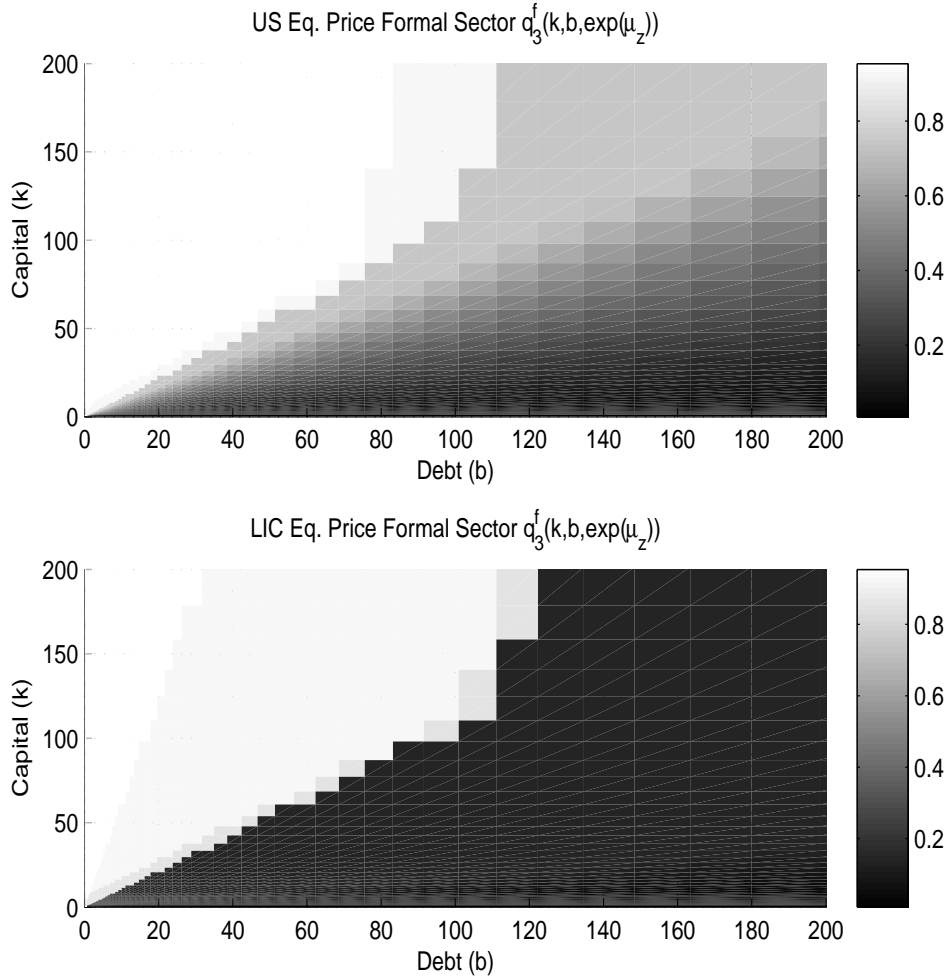


Figure 6: Equilibrium Prices in the Formal Sector across Countries

## 6.1 Understanding the role of each friction

In this section we evaluate the effects of each observed friction on the equilibrium outcomes one by one. We compute two imaginary economies that are used as benchmark for the experiments. First, we build a friction free economy (FF from now on), by setting  $(\lambda, \phi, \tau, c_\tau, \kappa) = (1, 0, 0, 0, 0)$ . This is an economy where the lenders recover 100% of their investment and there are no additional costs to enter the formal sector or to operate in it. Then, we build an economy with extremely high frictions (HF henceforth), so high that no firm

enters the formal sector. This economy is parameterized as  $(\lambda, \phi, \tau, c_\tau, \kappa) = (1, 0, 0, 0, \infty)$ . Note that when  $\kappa$  is high enough, the rest of the frictions are irrelevant, since there are no firms operating in the formal sector.

In order to understand the effects of each friction, we impose to the FF one friction at a time as shown in Table 7.

Table 7: Individual Friction Effects

Moment	FF	$\kappa$			$\tau$	$\lambda$		HF
		$\kappa_{us}$	$\kappa_{umic}$	$\kappa_{lic}$		$\lambda_{us}$	$\lambda_{lic}$	
Inf. Sector % GDP	0.00	1.21	8.54	28.97	0.00	0.00	0.00	100.00
Inf. Sector % Firms	0.00	11.74	57.58	95.02	0.00	0.00	0.00	100.00
Wage $w$	1.28	1.05	1.03	0.79	1.11	1.25	1.15	0.56
TFP $\equiv (Y/K^\alpha)/(Y_{us}/K_{us}^\alpha)$	1.13	0.98	0.98	0.85	1.10	1.13	1.08	0.98
Relative Output per Worker	1.28	1.05	1.04	0.79	1.11	1.26	1.15	0.56
Entry/Exit Rate	12.16	7.33	7.64	4.37	11.44	12.19	11.12	0.00
$b/k$ Formal Sector	0.62	0.49	0.45	0.47	0.60	0.68	0.65	-
$b/k$ Informal Sector	-	1.31	0.60	0.56	-	-	-	1.07
Avg. $n$ Formal Est.	5.35	22.68	21.89	125.38	7.88	5.39	8.52	-
Avg. $n$ Informal Est.	-	2.09	1.51	2.68	-	-	-	7.04
Total $K$ Formal Sector	3.50	2.85	2.61	1.31	2.35	3.36	2.63	0.00
Total $K$ Informal Sector	0	0.004	0.05	0.1	0.00	0	0	0.14

### 6.1.1 Entry costs

The effect of  $\kappa$  on total capital, informality, wages and output per worker follow standard intuition. Higher entry costs, generate a larger fraction of firms below the level at which it is optimal to switch to the formal sector. Once firms stay in the informal sector, firms face two constraints. First, they make explicit efforts not to be detected by the authorities by staying small in terms of capital. Second, they face tight borrowing constraints given by the fact that the recovery rate lenders obtain is zero. This generates a lower aggregate capital stock. The effect of  $\kappa$  on wages is through its impact on labor demand. A large fraction of small informal firms imply a low demand for workers, which lowers wages and makes the

formal firms inefficiently large in terms of workers.

Throughout the experiment firms substitute capital for workers. As  $\kappa$  increases, there is a selection effect in terms of productivity. Only the most productive firms find it profitable to enter the formal sector and operate without restrictions. Labor is redistributed towards these firms, which explains why output per worker falls much less than aggregate capital.

Measured total factor productivity follows a non-monotonic pattern as a function of  $\kappa$ . Given that the available technologies remain constant during the experiments, the effect on measured total factor productivity is only a consequence of distributional effects. For low and extremely high values of  $\kappa$ , total factor productivity is around 98% of the US. Only at high values of  $\kappa$ , as observed in the LIC, we see large distributional effects. Note that in the formal sector, firms are bigger than optimal in terms of employees, which is a consequence of lower wages and freely mobile labor.<sup>6</sup>

### 6.1.2 Recovery rates

If there is no fix cost to enter the formal sector, the entire economy becomes formal, independently of the value of  $\lambda$ . The recovery rate affects the schedule of prices  $q(\cdot)$ . Therefore borrowing and capital accumulation is affected as shown by the lower levels of aggregate capital. There is a fix point argument where changes in the recovery rate affect the prices  $q(\cdot)$  for a given default decision, but higher  $q(\cdot)$  generate stronger incentives to default.

As in the case of the entry cost, capital is substituted with labor (which is freely mobile), countering the effects of lower capital stocks in total output.

### 6.1.3 Profit taxes

Since taxes do not vary much across countries, we only compare the case of the friction free economy to the US. In this case, the value of the firm drops proportionally with the tax. It

---

<sup>6</sup>We also tried a case with a lower elasticity of substitution between capital and labor and the effect on wages and total factor productivity is much more pronounced.

is as if we lowered the firms productivity, therefore they demand less capital. The general equilibrium effect is one of lower wages, and through wages higher level of employment.

## 7 Conclusion

In this paper, we quantify the effects of differences across countries in the financial structure, as well as the cost of starting, operating and closing a business on wages, TFP and output per worker. We developed a general equilibrium firm dynamic model with endogenous entry and exit. Capital allocation is affected not only by their optimal investment and debt decisions, but also by the choice between operating in the formal or informal sector. Creating a formal sector firm involves the payment of a fixed cost and the obligation to pay taxes in the future. The main benefit of formality comes from access to better credit terms since lenders face a positive recovery after bankruptcy.

We find that if the average Low Income Country was to adopt the financial, tax and formal sector structure of the US, their wage would increase by 30.46%. We find that this increase in wages is not only a result of higher capital per worker but also one of higher total factor productivity. TFP would increase by 18.73% and output per worker by 30.25%. Consistent with Figure 1 we obtain a negative correlation between income per-worker and the size of the informal sector.

One of the main differences with previous papers in the literature is that we discipline the changes in parameters with the Doing Business Data Set. In particular, differences across countries are derived from their measure of recovery rates, cost of bankruptcy, tax structure and formal entry costs. We also restrict the ex-ante heterogeneity to be identical across countries, so differences in the ex-post distribution of firms are completely endogenous.

## 8 References

Alfaro, Laura, Charlton, Andrew and Kanczuk, Fabio (2007). “Firm-Size Distribution and Cross-Country Income Differences”. Mimeo. Harvard Business School, London School of Economics and Universidade de Sao Paulo.

Antunes, Antonio and Cavalcanti, Tiago (2007). “Start up costs, limited enforcement, and the hidden economy ”. *European Economic Review* 51: 203-224.

Castro, Rui, Clementi, Gian Luca and MacDonald, Glenn (2008). “Legal Institutions, Sectoral Heterogeneity, and Economic Development”. Forthcoming on the Review of Economic Studies.

D’Erasmus, Pablo (2007). “Investment and Firm Dynamics”. University of Maryland. Mimeo.

De Soto, Hernando (2000). “The Mystery of Capital”, New York: Basic Books.

Djankov, Simeon, Rafael La Porta, Florencio Lopez-De-Silanes, Andrei Shleifer, 2002. “The Regulation Of Entry” *The Quarterly Journal of Economics*, MIT Press, vol. 117(1), pages 1-37, February.

Djankov, Simeon, Hart, Oliver, McLiesh, Caralee and Shleifer, Andrei (2008). “Debt Enforcement Around the World” *Journal of Political Economy*, forthcoming.

Erosa, Andres and Hidalgo Cabrillana, Ana (2008). “On Finance as a Theory of TFP, Cross-Industry Productivity Differences, and Economic Rents”. Forthcoming at the *International Economic Review*.

Foster, Lucia, Haltiwanger, John and Krizan C. J. (2001). “Aggregate Productivity Growth: Lessons from Microeconomic Evidence”. In Charles R. Hulten, Edwin R. Dean, and Michael J. Harper (eds) “New Developments in Productivity Analysis”, Chicago, University of Chicago Press.

Hopenhayn, Hugo and Rogerson, Richard (1993). “Job Turnover and Policy Evaluation: A General Equilibrium Analysis”, *Journal of Political Economy* 101, 915-938.

Hsieh, Chang-Tai and Klenow, Peter (2007). “Misallocation and Manufacturing TFP in

China and India”, mimeo. University of California, Berkeley and Stanford University.

Moscoso Boedo, Hernan and Mukoyama, Toshihiko (2008). “Evaluating the Effects of Entry Regulations and Firing Costs on International Income Differences” University of Virginia mimeo.

Quintin, Erwan (2008). “Contract Enforcement and the Size of the Informal Economy”. *Economic Theory*, 37: 395-416.

Restuccia, Diego and Rogerson, Richard (2008). “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments”. *Review of Economic Dynamics*, Forthcoming.

Schneider, Friedrich and Enste, Dominik (2000). “Shadow Economies: Size, Causes, and Consequences”. *Journal of Economic Literature*, 38: 77-114.

Syverson, Chad, (2004), “Substitutibility and Product Dispersion”, *Review of Economics and Statistics*, 86 (2), 534-550.

Tauchen, George (1986). “Finite state Markov-chain approximation to univariate and vector autoregressions”. *Economics Letters* 20:177-81.

Tybout, James (2000). “Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?” *Journal of Economic Literature*, 38: 11-44.

## 9 Appendix 1: Stationary Distribution and Aggregates

At each point in time  $t$ , the economy is characterized by a mass of potential entrants  $M$  and a measure of firms  $\Gamma_t$  defined over the set  $S \equiv (z, k, b, \sigma_j, c_f)$  that includes the level of productivity, capital stock, debt stock, volatility and operative cost. The total measure of firms  $\Gamma_t(S)$  is composed by a mass of formal firms  $M\vartheta_t(S)$ , a mass of informal firms  $M\widehat{\vartheta}_t(S)$  and mass of newly formal firms  $M\widetilde{\vartheta}_t(S)$ . In particular,

$$\Gamma_t(S) = M(\vartheta_t(S) + \widehat{\vartheta}_t(S) + \widetilde{\vartheta}_t(S)).$$

The law of motion of  $\Gamma_t$  is given by  $H_t$ ,  $\Gamma_{t+1}(S_0) = H_t(\Gamma_t(S))$ . The mass of formal firms evolves from period  $t$  to  $t + 1$  according to transition probabilities  $\Upsilon_t^f(S)$  and  $\Upsilon_t^n(S)$  as follows:

$$\vartheta_{t+1}(S_0) = \int_{S_0} \left[ \int_S \Upsilon_t^f(S) \vartheta_t(dS) + \int_S \Upsilon_t^n(S) \widetilde{\vartheta}_t(dS) \right] dS_0$$

where the first term correspond to formal firms in period  $t$  that do not exit and the second term is associated with newly formal firms that stay in the market. More specifically, the transition probabilities are given by

$$\begin{aligned} \Upsilon_t^f(S) &= \eta_j(z'|z)\xi(c'_f)I\{k' = k_j^{ff}(z, k, b, c_f), b' = b_j^{ff}(z, k, b, c_f), 0 = \chi_j^f(z, k', b', c'_f)\}, \\ \Upsilon_t^n(S) &= \eta_j(z'|z)\xi(c'_f)I\{k' = \widetilde{k}_j^f(z, k, b, c_f), b' = \widetilde{b}_j^f(z, k, b, c_f), 0 = \chi_j^f(z, k', b', c'_f)\}, \end{aligned}$$

where  $I\{\cdot\}$  corresponds to the indicator function. Similarly, the evolution of the mass of informal firms can be written explicitly as

$$\widehat{\vartheta}_{t+1}(S_0) = \int_{S_0} \left[ \int_S \Upsilon_t^i(S) \widehat{\vartheta}_t(dS) \right] dS_0 + \int \Upsilon_t^e(\sigma_j, z_0) d\mu(\sigma_j) d\nu(z_0)$$

where the transition probabilities  $\Upsilon_t^i(S)$  and  $\Upsilon_t^e$  correspond to informal firms that choose to stay informal and to newly born firms created in the informal sector correspondingly. They can be written explicitly as

$$\begin{aligned} \Upsilon_t^i(S) &= [1 - \omega(k')] \eta_j(z'|z)\xi(c'_f)I\{k' = k_j^{ii}(z, k, b, c_f), b' = b_j^{ii}(z, k, b, c_f), 0 = \chi_j^i(z, k', b', c'_f)\}, \\ \Upsilon_t^e(\sigma_j, z_0) &= I\{0 = \chi_j^i(z_0, 0, 0, 0)\}. \end{aligned}$$

Finally, the evolution of the mass of new formal firms can be written explicitly as:

$$\widetilde{\vartheta}_{t+1}(S_0) = \int_{S_0} \left[ \int_S \widetilde{\Upsilon}_t^i(S) \widehat{\vartheta}_t(dS) \right] dS_0 + \int \widetilde{\Upsilon}_t^e(\sigma_j, z_0) d\mu(\sigma_j) d\nu(z_0)$$

where the transition probabilities  $\widetilde{\Upsilon}_t^i(S)$  and  $\widetilde{\Upsilon}_t^e$  correspond to informal firms that choose switch to formality and to newly born firms created in the formal sector correspondingly.

They can be written explicitly as

$$\begin{aligned} \tilde{\Upsilon}_t^i(S) = & [1 - \omega(k')] \eta_j(z'|z) \xi(c'_f) I\{k' = \tilde{k}_j^i(z, k, b, c_f), b' = \tilde{b}_j^i(z, k, b, c_f), 2 = \chi_j^i(z, k', b', c'_f)\} \\ & + \omega(k') \eta_j(z'|z) \xi(c'_f) I\{k' = \tilde{k}_j^i(z, k, b, c_f), b' = \tilde{b}_j^i(z, k, b, c_f), 0 = \tilde{\chi}_j^i(z, k', b', c'_f)\} \quad , \end{aligned}$$

$$\tilde{\Upsilon}_t^e(\sigma_j, z_0) = I\{2 = \chi_j^i(z_0, 0, 0, 0)\}.$$

This paper focuses on the study of the invariant distribution of firms that is the fixed point of the transition function  $H$ , i.e.  $\Gamma^* = H(\Gamma^*)$ .<sup>7</sup> In this equilibrium aggregate profits are

$$\Pi = M \left( \int d_j^f(z, k, b, c_f) \vartheta(dS) + \int d_j^i(z, k, b, c_f) \hat{\vartheta}(dS) + \int \tilde{d}_j^f(z, k, b, c_f) \tilde{\vartheta}(dS) \right).$$

The tax transfer is given by

$$\begin{aligned} T = & M \left[ \int \tau [zk^\alpha n(z, k)^\gamma - c_f - w(n(z, k) + c_\tau)] \vartheta(dS) + \right. \\ & \left. \int \tau [zk^\alpha n(z, k)^\gamma - c_f - w(n(z, k) + c_\tau + \kappa)] \tilde{\vartheta}(dS) \right]. \end{aligned}$$

The total entry cost is  $E = Mc_e$  and the aggregate exit value of firms is given by

$$\begin{aligned} X = & M \int \left[ \int_S [(p^f k - b) I\{2 = \chi_j^f(z, k, b, c'_f)\} \right. \\ & + \max\{0, p^f k(1 - \phi) - \lambda b\} I\{1 = \chi_j^f(z, k, b, c'_f)\}] d \left[ \vartheta(S) + \tilde{\vartheta}_t(S) \right] \\ & \left. + \int_S \left[ \frac{[1 - \omega(k)] p^i k I\{1 = \chi_j^i(z, k, b, c'_f)\}}{+\omega(k) \max(p^i k - w\kappa, 0) I\{1 = \tilde{\chi}_j^i(z, k, b, c'_f)\}} \right] d \hat{\vartheta}(S) \right] d \xi(c'_f) \end{aligned}$$

The condition to clear the labor markets is:

$$1 = M \left( \int_S n(z, k) \vartheta(dS) + \int_S n(z, k) \hat{\vartheta}(dS) + \int_S n(z, k) \tilde{\vartheta}(dS) \right),$$

where  $n(z, k)$  denotes the employment choice, both in the formal and informal sectors. In particular, note that, for any firm, the optimal level of labor input solves the following problem:

$$\max_n \left\{ zk^\alpha n^\gamma - wn \right\} \quad (1)$$

---

<sup>7</sup>Standard arguments can be used to show the existence of a stationary measure of establishments and the homogeneity of degree 1 in the mass of entrants. Moreover, it can also be shown that an equilibrium with no entry and exit also exists; however, in what follows, we only discuss the relevant equilibrium i.e. one with  $M > 0$ .

The solution at state  $(z, k)$  is:

$$n(z, k) = \left[ \frac{\gamma z}{w} \right]^{1/(1-\gamma)} k^{\alpha/(1-\gamma)}.$$

Let  $a(z) = (z/w^\gamma)^{\frac{1}{1-\gamma}} \left[ \gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right]$  and  $\theta = \frac{\alpha}{1-\gamma}$ . Then, by evaluating expression (1) at  $n(z, k)$ , we obtain:

$$zk^\alpha n(z, k)^\gamma - wn(z, k) = a(z)k^\theta. \quad (2)$$

## 9.1 Definition of equilibrium

A stationary competitive equilibrium is a set of functions  $\{W^f, W^i, V^f, V^i, h^f, h^i, g^f, g^i, \chi^f, \chi^i, q^f, q^i\}$ , a wage rate  $w$ , aggregate distributions of firms  $\vartheta$  and  $\widehat{\vartheta}$ , a mass of entrants  $M$  such that:

1. Given prices, the value function of the firms and the policy functions are consistent with firms optimization.
2. The free entry condition is satisfied:  $W_e = 0$ .
3. Lenders make zero profit for every type of loan.
4. Invariant distributions:  $\vartheta$  and  $\widehat{\vartheta}$  are a fixed point of the mappings  $H$  and  $\widehat{H}$ .
5. Aggregate consumption:  $C = w + \Pi + T - E + X$ .
6. The labor market clears:

$$1 = M \left( \int_S n(z, k) \vartheta(dS) + \int_S n(z, k) \widehat{\vartheta}(dS) + \int_S n(z, k) \widetilde{\vartheta}(dS) \right),$$

## 9.2 Computation Algorithm

The model is solved numerically. Computation of the equilibrium requires three steps: an inner loop, where the decision problem of the firms given prices is solved; a middle loop, where the schedule of loan prices is obtained; and an outside loop where the wage rate and the mass of entrant adjust to clear the labor market and to satisfy the free entry condition. To solve the model we use the discrete state space method.

For the US, start with  $(\lambda, \phi, \tau, c_\tau, \kappa)_{US}$ , set  $w = 1$  and make an initial guess for the vectors  $q_j^f(k', b', z)$  and  $q_j^i(k', b', z)$ . Iterate on  $q_j^f(k', b', z)$  and  $q_j^i(k', b', z)$  until the lender makes zero profit on each contract. Adjust the mass of potential entrants until the labor market clears. Obtain  $c_e$  for the US. For the rest of the countries, adjust the country specific parameters  $(\lambda, \phi, \tau, c_\tau, \kappa)$ , and iterate on  $w$  and  $q_j^f(k', b', z)$  and  $q_j^i(k', b', z)$  until the lender makes zero profits and the  $c_e$  across countries converges to the one obtained for the US case. Finally adjust the mass of potential entrants until labor markets clear.

## 10 Appendix 2: International Data

Here we present the frictions  $(\lambda, \phi, \tau, c_\tau, \kappa)$  for each country in the dataset together with the income group to which it belongs according to the World Bank (H = high income, UM = upper middle income, LM = lower middle income and L = low income)

		$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$			$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$
AFG	L	0.00	0.00	0.00	0.13	150.74	CRI	UM	0.25	0.15	0.20	0.14	3.01
AGO	LM	0.10	0.22	0.25	0.13	11.36	CZE	UM	0.21	0.15	0.06	0.45	0.80
ALB	LM	0.00	0.00	0.17	0.12	5.54	DEU	H	0.52	0.08	0.23	0.09	1.00
ARE	H	0.10	0.30	0.00	0.01	0.54	DJI	LM	0.16	0.18	0.18	0.05	12.47
ARG	UM	0.30	0.12	0.03	0.22	2.94	DMA	UM	0.00	0.00	0.26	0.06	0.92
ARM	LM	0.42	0.04	0.12	0.46	0.68	DNK	H	0.87	0.04	0.24	0.06	0.81
ATG	H	0.36	0.07	0.31	0.10	0.86	DOM	LM	0.09	0.38	0.21	0.23	1.76
AUS	H	0.79	0.08	0.27	0.05	0.75	DZA	LM	0.42	0.07	0.08	0.22	1.30
AUT	H	0.72	0.18	0.15	0.08	1.36	ECU	LM	0.16	0.18	0.19	0.29	3.71
AZE	LM	0.30	0.08	0.14	0.18	5.87	EGY	LM	0.17	0.22	0.14	0.34	4.65
BDI	L	0.00	0.00	0.18	0.07	88.48	ERI	L	0.00	0.00	0.09	0.10	1.25
BEL	H	0.86	0.04	0.06	0.08	1.18	ESP	H	0.73	0.15	0.22	0.11	1.54
BEN	L	0.17	0.22	0.17	0.13	6.20	EST	UM	0.38	0.09	0.09	0.04	0.63
BFA	L	0.22	0.09	0.15	0.13	7.03	ETH	L	0.32	0.15	0.27	0.10	8.60
BGD	L	0.23	0.08	0.27	0.15	8.49	FIN	H	0.87	0.04	0.17	0.13	1.34
BGR	LM	0.32	0.09	0.04	0.30	5.47	FJI	LM	0.20	0.38	0.31	0.07	1.26
BHR	H	0.63	0.10	0.00	0.02	0.76	FRA	H	0.45	0.09	0.08	0.06	0.64
BHS	H	0.55	0.04	0.00	0.03	3.14	FSM	LM	0.04	0.38	0.52	0.06	1.81
BIH	LM	0.36	0.09	0.22	0.21	7.95	GAB	UM	0.15	0.15	0.20	0.13	1.33
BLR	LM	0.33	0.22	0.22	0.57	1.13	GBR	H	0.84	0.06	0.21	0.05	1.08
BLZ	UM	0.63	0.23	0.20	0.07	0.99	GEO	LM	0.28	0.04	0.14	0.19	0.56
BOL	LM	0.37	0.15	0.00	0.52	3.16	GHA	L	0.24	0.22	0.18	0.11	13.85
BRA	LM	0.17	0.12	0.21	1.25	2.09	GIN	L	0.22	0.08	0.22	0.20	4.60
BRN	H	0.47	0.04	0.32	0.07	0.92	GMB	L	..	0.15	0.41	0.18	6.96
BTN	L	0.00	0.00	0.34	0.13	2.30	GNB	L	0.00	0.00	0.15	0.10	29.96
BWA	UM	0.60	0.15	0.17	0.07	3.81	GNQ	UM	0.00	0.00	0.14	0.14	3.53
CAF	L	0.00	0.76	1.77	0.24	5.81	GRC	H	0.44	0.09	0.14	0.11	1.08
CAN	H	0.89	0.04	0.26	0.06	1.26	GRD	UM	0.00	0.00	0.28	0.07	1.08
CHE	H	0.47	0.04	0.09	0.03	1.02	GTM	LM	0.28	0.15	0.02	0.17	13.21
CHL	UM	0.21	0.15	0.18	0.15	1.59	GUY	LM	0.18	0.29	0.27	0.14	3.72
CHN	LM	0.35	0.22	0.12	0.24	8.10	HKG	H	0.80	0.09	0.19	0.04	0.56
CIV	L	0.34	0.18	0.10	0.13	5.61	HND	LM	0.21	0.15	0.27	0.11	5.57
CMR	LM	0.26	0.15	0.28	0.67	15.41	HRV	UM	0.31	0.15	0.11	0.09	7.90
COD	L	0.05	0.29	0.00	0.15	22.92	HTI	L	0.03	0.30	0.23	0.08	12.11
COG	LM	0.20	0.24	0.00	0.29	5.08	HUN	UM	0.38	0.15	0.09	0.16	0.76
COL	LM	0.53	0.01	0.18	0.12	7.17	IDN	LM	0.14	0.18	0.27	0.13	3.68
COM	L	0.00	0.00	0.27	0.05	3.18	IND	L	0.10	0.09	0.23	0.13	5.54
CPV	LM	0.00	0.00	0.22	0.05	7.22	IRL	H	0.87	0.09	0.14	0.04	0.99

		$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$			$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$
IRN	LM	0.23	0.09	0.18	0.17	7.15	NLD	H	0.83	0.04	0.22	0.09	1.84
IRQ	LM	0.00	0.00	0.11	0.15	11.46	NOR	H	0.89	0.01	0.24	0.04	1.20
ISL	H	0.77	0.04	0.09	0.07	0.44	NPL	L	0.25	0.09	0.20	0.20	4.33
ISR	H	0.45	0.23	0.26	0.11	1.91	NZL	H	0.76	0.04	0.32	0.03	0.44
ITA	H	0.57	0.22	0.28	0.16	2.28	OMN	UM	0.35	0.04	0.10	0.03	7.95
JAM	LM	0.65	0.18	0.29	0.20	4.49	PAK	L	0.39	0.04	0.14	0.27	8.14
JOR	LM	0.27	0.09	0.15	0.05	5.41	PAN	UM	0.32	0.18	0.17	0.23	1.82
JPN	H	0.93	0.04	0.34	0.17	0.84	PER	LM	0.25	0.07	0.27	0.20	2.41
KAZ	LM	0.25	0.18	0.16	0.13	15.06	PHL	LM	0.04	0.38	0.26	0.09	1.90
KEN	L	0.32	0.22	0.33	0.20	1.22	PLW	UM	0.38	0.23	0.00	0.06	0.51
KGZ	L	0.14	0.15	0.03	0.10	4.61	PNG	L	0.25	0.23	0.22	0.09	1.93
KHM	L	0.00	0.00	0.19	0.07	4.34	POL	UM	0.30	0.20	0.13	0.20	2.49
KIR	LM	0.00	0.00	0.23	0.06	8.32	PRI	H	0.55	0.08	0.25	0.10	6.11
KNA	UM	0.00	0.00	0.33	0.08	0.48	PRT	H	0.69	0.09	0.14	0.16	1.48
KOR	H	0.81	0.04	0.19	0.12	1.85	PRY	LM	0.21	0.09	0.10	0.16	4.99
KWT	H	0.35	0.01	0.04	0.06	2.11	PSE	LM	0.00	0.00	0.16	0.07	15.37
LAO	L	0.00	0.00	0.25	0.27	2.62	QAT	H	0.53	0.22	0.00	0.02	0.32
LBN	UM	0.19	0.22	0.12	0.09	3.66	ROM	UM	0.30	0.09	0.10	0.10	1.64
LBR	L	0.08	0.43	0.25	0.08	611.84	RUS	UM	0.28	0.09	0.11	0.22	28.16
LCA	UM	0.43	0.09	0.26	0.03	0.96	RWA	L	0.00	0.00	0.20	0.08	7.77
LKA	LM	0.43	0.05	0.27	0.12	15.63	SAU	H	0.38	0.22	0.02	0.04	1.27
LSO	LM	0.34	0.08	0.15	0.16	10.31	SDN	L	0.00	0.00	0.09	0.09	3.76
LTU	UM	0.48	0.07	0.08	0.08	1.64	SEN	L	0.32	0.07	0.15	0.32	6.64
LUX	H	0.42	0.15	0.04	0.03	0.93	SGP	H	0.91	0.01	0.08	0.04	0.33
LVA	UM	0.29	0.13	0.02	0.13	0.79	SLB	L	0.24	0.38	0.25	0.04	5.57
MAR	LM	0.35	0.18	0.21	0.17	3.51	SLE	L	0.09	0.42	0.00	0.19	5.91
MDA	LM	0.29	0.09	0.10	0.11	2.35	SLV	LM	0.31	0.09	0.17	0.15	2.73
MDG	L	0.00	0.00	0.20	0.11	8.26	SRB	LM	0.25	0.23	0.12	0.13	22.68
MDV	LM	0.18	0.04	0.00	0.00	0.73	STP	L	0.00	0.00	0.36	0.20	9.39
MEX	UM	0.64	0.18	0.23	0.26	1.89	SUR	LM	0.08	0.30	0.28	0.10	5.39
MHL	LM	0.18	0.38	0.00	0.06	0.73	SVK	UM	0.46	0.18	0.07	0.16	0.99
MKD	LM	0.17	0.28	0.14	0.04	19.23	SVN	H	0.46	0.08	0.13	0.13	1.74
MLI	L	0.21	0.18	0.13	0.13	13.72	SWE	H	0.75	0.09	0.17	0.06	1.40
MNE	LM	0.44	0.08	0.10	0.18	14.01	SWZ	LM	0.35	0.15	0.28	0.05	1.72
MNG	L	0.22	0.08	0.07	0.10	1.48	SYC	UM	0.00	0.00	0.24	0.04	1.05
MOZ	L	0.15	0.09	0.28	0.11	8.82	SYR	LM	0.30	0.09	0.23	0.16	7.55
MRT	L	0.07	0.09	0.00	0.33	5.69	TCD	L	0.00	0.00	0.31	0.06	12.20
MUS	UM	0.34	0.15	0.11	0.08	0.77	TGO	L	0.27	0.15	0.13	0.13	17.42
MWI	L	0.15	0.30	0.30	0.14	14.84	THA	LM	0.42	0.36	0.29	0.13	0.66
MYS	UM	0.39	0.15	0.17	0.07	0.98	TJK	L	0.25	0.09	0.18	0.11	15.58
NAM	LM	0.40	0.15	0.17	0.18	2.60	TMP	L	0.00	0.00	0.28	0.31	1.49
NER	L	0.14	0.18	0.15	0.13	29.42	TON	LM	0.25	0.22	0.26	0.08	4.09
NGA	L	0.28	0.22	0.22	0.45	8.50	TTO	UM	0.00	0.00	0.22	0.05	0.90
NIC	LM	0.34	0.15	0.25	0.12	10.58	TUN	LM	0.52	0.07	0.12	0.11	10.52

		$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$			$\lambda$	$\phi$	$\tau$	$c_\tau$	$\kappa$
TUR	UM	0.20	0.15	0.17	0.11	3.17	VEN	UM	0.06	0.38	0.10	0.42	5.18
TWN	H	0.81	0.04	0.20	0.16	1.92	VNM	L	0.18	0.15	0.21	0.50	3.97
TZA	L	0.21	0.22	0.20	0.08	22.21	VUT	LM	0.41	0.38	0.00	0.06	4.36
UGA	L	0.41	0.30	0.22	0.11	8.50	WSM	LM	0.14	0.38	0.12	0.11	1.64
UKR	LM	0.09	0.42	0.12	0.41	20.44	YEM	L	0.29	0.08	0.35	0.12	3.16
URY	UM	0.43	0.07	0.23	0.16	2.28	ZAF	UM	0.32	0.18	0.25	0.10	0.87
USA	H	0.77	0.07	0.24	0.09	0.26	ZMB	L	0.30	0.09	0.02	0.06	11.26
UZB	L	0.19	0.10	0.02	0.09	2.09	ZWE	L	0.00	0.22	0.00	0.12	172.18
VCT	UM	0.00	0.00	0.35	0.06	0.59							